AD-753 493

METHODOLOGY FOR EVALUATION OF TASK FORCE DEFENSE CONCEPTS. EFFECTIVENESS OF CTG (CARRIER TASK GROUP) ACTIVE AIR DEFENSE AGAINST AIR RECONNAISSANCE

Anthony Kooharian, et al

Operations Research, Incorporated

Prepared for:

Office of Naval Research

29 September 1972

AD753493

Technical Report No. 748

Methodology for Evaluation of Task Force Defense Concepts

Effectiveness of CTG Active Air Defense Against Air Reconnaissance

by

Anthony Kooharian and Henry A. Young

29 September 1972

Prepared under Contract No. N00014-70-C-0419 (NR 274-119)
for Naval Analysis Programs (Code 462)
Office of Naval Research
Department of the Navy
Arlington, Virginia 22217

OPERATIONS RESEARCH, Inc.

SILVER SPRING, MARYLAND

Methodology for Evaluation of Task Force Defense Concepts

Effectiveness of CTG Active Air Defense Against Air Reconnaissance

by

Anthony Kooharian and Henry A. Young

29 September 1972

Prepared under Contract No. N00014-70-C-0419 (NR 274-119)
for Naval Analysis Programs (Code 462)
Office of Naval Research
Department of the Navy
Arlington, Virginia 22217

I-a

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Operations Research, Inc. 1400 Spring Street Silver Spring, Maryland 20910	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE
METHODOLOGY FOR EVALUATION OF TASK FORCE DEFENSE CONCEPTS (U)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Analytical Methodology Development June 1971 - 30 September 1972

5. AUTHOR(S) (First name, middle initial, last name)
Henry A. Young
Anthony Kooharian

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
29 September 1972	58	1

8a. CONTRACT OR GRANT NO. N00014-70-C-0419 b. PROJECT NO. RF018-02-06 c. NR274-119 d.	9a. ORIGINATOR'S REPORT NUMBER(S) ORI TR 748 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release, distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Analysis Programs (Code 462) Office of Naval Research Arlington, Virginia 22217

13. ABSTRACT

This is a detailed treatment of that portion of the Carrier Task Group (CTG) air defense problem that deals with denial of enemy air reconnaissance effort. The enemy is burdened in their attack with the requirement to conduct sufficient air reconnaissance in order to mount an adequate attack. The study concentrates on the evaluation of the effectiveness of an active CTG air defense strategy against the air reconnaissance threat. This signifies that enemy aircraft approaching the CTG operating area can detect radiation first and can plan their tactics accordingly while suspressing their own electromagnetic radiation until absolutely required.

The air reconnaissance active defense methodology developed in the study consists essentially of three models:

- Air barrier engagement model
- Air barrier penetration model
- Air barrier logistics model.

I-i

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Task Force Defense						
Fleet Air Defense						
Air Reconnaissance Defense						
Air Defense Barrier Analysis						
Air Barrier Engagement Modeling						
Air Barrier Penetration Modeling						
Air Barrier Logistics Modeling						
Air to Air Engagement Analysis						

UNCLASSIFIED

EXECUTIVE SUMMARY

This study forms a portion of a continuing investigation into the development of methodology for evaluation of task force defense concepts. The work has been performed by Operations Research, Inc. (ORI) over the past several years and is under the sponsorship of the Office of Naval Research, Naval Analysis Programs (Code 462).

The long-term objective of this work is to develop a unified methodology that permits the weapons systems planner to evaluate the effectiveness of the various elements of fleet defense. Interim results have been reported previously that represent an initial formulation of such a unified methodology. This has been accomplished in the context of examining the effectiveness of fleet defense against the threat of anti-ship missiles, both air-launched and submarine-launched. Two analytical models of Carrier Task Group (CTG) functional defense effectiveness have been developed. Both of these models are extremely flexible and adaptable to a wide range of tactical, operational and technical considerations. One model measures CTG defense effectiveness against a self-contained submarine reconnaissance/attack threat. The other model measures CTG defense effectiveness against a self-contained air reconnaissance/attack threat. "Self-contained" means "without assistance from other friendly units". This definition applies here and elsewhere.

The present study is a more detailed treatment of that portion of the CTG air defense problem that deals with denial of enemy air reconnaissance efforts. Previous work has assumed that the enemy forces have localized the High Value Unit (HVU) to one of a number of credible formations positioned within a large CTG operating area. The present study drops this assumption and burdens the enemy with the requirement to conduct sufficient air reconnaissance in order to mount an adequate attack. The study concentrates on evaluation of the effectiveness of an <u>active</u> CTG air defense strategy against the air reconnaissance threat. This signifies that enemy aircraft approaching

the CTG operating area can detect AEW/CAP radiations first and can plan their tactics accordingly while suppressing their own electromagnetic radiations until absolutely required.

The air reconnaissance active defense methodology developed in the study consists essentially of three distinct analytical models, linked to one another by the defense functional parameters they share in common. These models are:

- Air barrier engagement model, which characterizes barrier geometry, enemy penetration tactics and attack engagement probabilities
- Air barrier penetration model, which characterizes barrier sector fighting strength, and enemy total barrier penetration both by engaged and non-engaged raid groups
- Air barrier logistics model, which characterizes the logistic requirements to maintain a barrier of given geometry and strength.

The air barrier engagement model expresses results in terms of barrier engagement probabilities as a function of the major parameters involved. Results of the penetration model are expressed in terms of the total number of penetrating reconnaissance aircraft by force size, penetration tactics, barrier strength and number of escort fighters. The air barrier logistics model results are presented in terms of the number of aircraft required to patrol barriers at a selected range from the carrier as a function of their surveillance capability, maintainability and cruise range characteristics.

TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

I. BACKGROUND DISCUSSION

CARRIER TASK GROUP DEFENSE STUDY

The Carrier Task Group (CTG) Defense Study, [1/] of which this investigation is a part, seeks to develop analytical methodology of use in the analysis of overall CTG defense effectiveness against anti-ship cruise missile threats. The methodologies developed emphasize the integration of the combined CTG capabilities for defense against enemy

- Target intelligence acquisition
- Attack vehicle positioning
- Missile penetration.

This study of the active defense of a CTG objective area against enemy air reconnaissance complements the air defense effectiveness analysis given in an earlier project interim report. [2/] The earlier air defense analysis quantitatively explored the utility, to the CTG defense, of creating ambiguity in enemy bomber targeting intelligence. It was assumed that enemy strategic intelligence activities, of an unspecified nature, had localized a small number of credible formation targets, only one of which contained a vital (high-value) unit (HVU). A methodology for analyzing the consequences of following various multi-wave reconnaissance and/or attack strategies was developed and exercised parametrically. A general conclusion of that study was that CTG overall air defense effectiveness could be significantly enhanced

[1/] Study sponsored by the Naval Analysis Programs Office, Office of Naval Research, Department of the Navy, Contract No. N00014-70-C-0419.

[2/] A Methodology for Evaluation of Task Force Defense Concepts (U), ORI Report, 5 November 1971, CONFIDENTIAL.

by masking one HVU formation amongst even a small number of credible targets. In particular, targeting ambiguity increases the size of the enemy force needed to attack the HVU successfully. Furthermore the required strength of the cruise-missile intercept capability of the HVU formation could be substantially reduced and still expect to maintain the same level of HVU survival probability.

The following analysis complements this investigation by dropping the assumption that the enemy has localized any targets. The intent is to measure the added burden on the enemy when CTG targets, positioned somewhere in a large objective area, have not been localized and air reconnaissance must find these. It is intended that the output of this strategic air reconnaissance defense model provide the input to the earlier air defense model. In addition, the air reconnaissance defense model is directly applicable to the analysis of the combined air-assisted/submarine-launched cruise -missile threat.

This report concentrates on the effectiveness of active air defense against the air reconnaissance threat. The tactical significance of "active defense" is that enemy reconnaissance aircraft approaching the defended CTG objective area can detect AEW/CAP radiations first and can plan tactics of defense avoidance, roll-back, or saturation while suppressing their own electromagnetic radiation until absolutely required.

THE AIR RECONNAISSANCE DEFENSE PROBLEM

Introduction

The purpose of a defense against enemy air reconnaissance of the CTG objective area is to force the enemy to expend resources to obtain the timely target location and classification data necessary for the most effective use of bomber (or submarine) attack forces. With imperfect prior target intelligence, attack forces themselves, must first search for suitable CTG targets and thereby expose themselves to counter-action while operating in a more vulnerable, and less lethal, reconnaissance mode. For example, if a force of attack bombers must first penetrate a large defended area and then use radar to search for targets of opportunity, their concentration against a dispersed target set is disrupted; their location is continuously revealed; there is little time to classify targets once detected; and sub-optimum approaches to fire-control system lock-on and weapon release lines may be required. Consequently, prior air reconnaissance of the objective area to locate and, if possible, classify targets and designate these to a follow-on bomber force (or in-place submarine force) is indicated. This study is concerned with the effectiveness of an active defense against such air reconnaissance attacks.

Figure 1 illustrates the structure of this air reconnaissance defense problem. The CTG (BLUE) has dispersed several real, or decoy, target formations in an objective area some hundreds of miles in diameter. The enemy (RED) must

- Fly a force of reconnaissance (recce) aircraft perhaps accompanied by fighter escort, to the objective area
- Penetrate an active air defense barrier line with sufficient aircraft to locate and, if possible, classify BLUE targets
- Deliver this target intelligence data back to home base, or to a bomber force trailing the reconnaissance attack by a distance short enough to prevent any significant degradation of the intelligence before their arrival or to a ship/sub attack force.

Thus, BLUE must balance his commitment to a strong airborne reconnaissance defense against the need for a reserve of AEW and fighter aircraft to meet potential follow-on bomber attacks.

There are essentially three major components to this air defense problem, the characteristics of which must be represented in the analytic functional defense effectiveness model. These are

- <u>BLUE CTG RECCE DEFENSE DISPOSITION</u>
  BLUE's selection and occupation of an area of strategic uncertainty large enough to present RED with an air reconnaissance problem; the specification of an air defense barrier relative to this objective area; and the determination of a CTG disposition, EMCON, and ECM posture.
- <u>RED RECCE THREAT</u>
  RED's choice of the number of reconnaissance and escort fighter aircraft to commit, and a barrier penetration tactic.
- <u>BLUE INNER DEFENSES</u>
  BLUE's reserve defensive response to barrier detections and penetrations, and the initiation of RED radar surveillance.

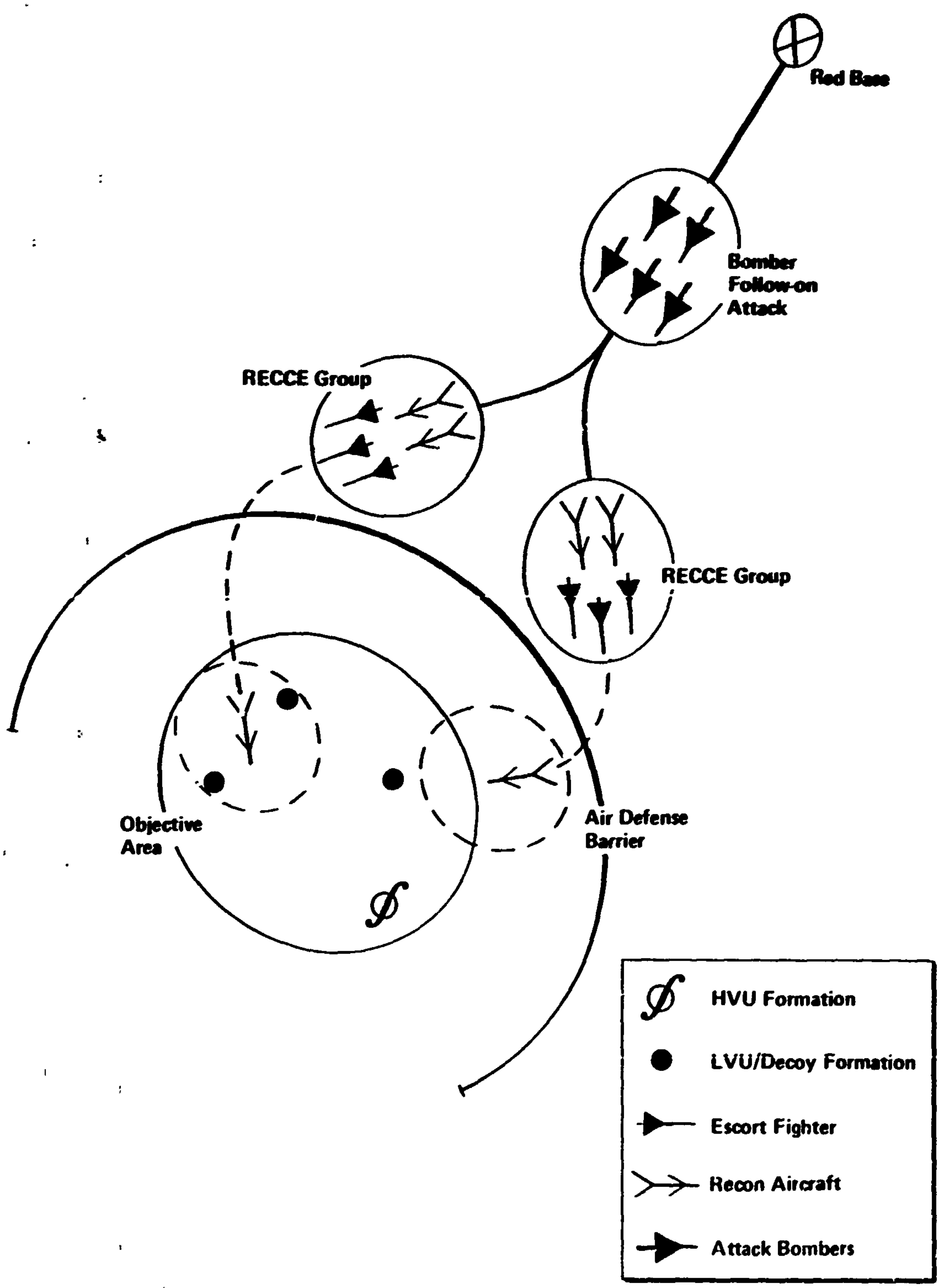


FIGURE 1. SCHEMATIC ILLUSTRATION OF THE AIR RECONNAISSANCE DEFENSE PROBLEM

Air Defense Barrier

The more extensive the battle space BLUE can actively control (the larger the area RED must search), or the more firepower that BLUE can bring to bear at any point in this battle-space, then the better BLUE's defense. However, the limited number and "reach" of each CTG air or surface-based surveillance and weapon delivery system, and the inverse relation between aircraft range and useful payload, restricts BLUE's ability to effectively control surrounding battle space.

In the earlier air battle model it was assumed that BLUE had a perfect capability for battle-space surveillance, raid diagnosis, and interceptor control. For the case of air attacks directed at known target locations, against a pre-positioned close-in BLUE defense, this assumption is a useful first step. However, the requirement to prevent a multi-axis RED reconnaissance threat from mapping any part of a large objective area means that the effectiveness of the defense rests upon the CTG's ability to control an extended battle space with a limited number of defensive platforms of finite reach. This is the theme around which the air reconnaissance defense model is organized.

The analytical air reconnaissance active defense model developed in this study is a functional characterization of such a defense that applies to many different physical means of producing the functional capabilities assumed. For example, the barrier is functionally described by its length, range from the center of the objective area; number of defense sectors (stations) along the barrier; the fraction of each sector that can be effectively controlled; and the firepower available in each sector. The primary interpretation of the functional model is that each barrier station is manned by one AEW aircraft and a certain number (including zero) of CAP interceptors. The defensive strength of a barrier sector is measured by the expected number of lethal shots (or passes) that can be made against a detected recce raid group. However, a hypothetical AEW with self-contained AAM firing capability, or an AEW surveillance platform capable of guiding ship launched SAMs against attacking aircraft, are other defensive means that can be characterized by the methodology.

The width of each defense sector that can be effectively controlled reflects the influence of such operational variables as the detection and vector capabilities of EW/CAP aircraft; target cross-section, speed and altitude; jamming and natural environments. Barrier range is influenced by the requirement to protect an objective area large enough to make it useful for RED to

conduct a prior reconnaissance attack before committing his bomber (or submarine) force, and far enough from this area to engage reconnaissance aircraft at, or beyond, their target location and/or classification range. The barrier range (along with the deck-turn-around time and availability of BLUE aircraft) is a central factor in determining the logistic back-up needed per aircraft continuously on station. These data multiplied by the number of barrier stations, and number of aircraft per station, specify the total number of AEW/VF aircraft required to support the barrier. A further important requirement on the barrier force is that the CV be free to move anywhere in the objective area while maintaining the barrier from a position offset from the center of the objective area. Then, RED cannot infer CV location from an examination of the position of the barrier.

If barrier aircraft do not radiate, RED aircraft could penetrate silently past the barrier, position one or more reconnaissance aircraft over the objective area, map this, and possibly attack targets opportunistically 3/. Consequently, BLUE barrier aircraft must actively radiate to detect penetrating aircraft. However, if the barrier is "loose" in the sense that the total length of the barrier is not effectively controlled by the number of barrier stations, it is assumed that intermittent radar use, AEW/CAP motions, and possibly changes in radiated power, can effectively hide the "holes" between barrier stations from RED ELINT techniques. Consequently, RED must plan to penetrate the barrier without knowledge of the precise location of instantaneously uncovered portions of the barrier. This assumption is reasonable when the barrier is not too loose, and it provides a means for BLUE to reduce the barrier logistics requirement, or increase the strength of a smaller number of stations.

The barrier length is determined by the need to deny RED the option of an end-run around the barrier. This parameter reflects the effect of several operational parameters such as RED aircraft range, distance to the RED air base, re-fueling capabilities, and land-sea political geography.

Reconnaissance Force Size and Penetration Tactics

RED must select reconnaissance, and fighter escort, force sizes, and a barrier penetration strategy. The choice of force size is conditioned by the

- Reconnaissance requirement, determined by the number of aircraft RED believes necessary to conduct an effective reconnaissance of the objective area after barrier penetration

---

3/ A purely passive defense of the objective area that forces RED to make the first radiation move is under study. If feasible, the structure of such a defense will differ from the active defense analyzed here.

- Barrier geometry and defensive strength in each sector
- Optimal barrier penetration tactics.

The reconnaissance requirement is a complex function of RED radar capabilities and scanning pattern; reconnaissance altitude and speed; BLUE active and passive countermeasures; environmental variables; and the surveillance time available before a BLUE counter-attack on a radiating aircraft arrives. If RED seeks reliable target classification intelligence, then, in the absence of characteristic ELINT radiation from the CTG targets, the radar surveillance range may be sharply reduced and a modest number of recce aircraft may be needed to map a large objective area. If RED is content with target location data only, and willing to risk attacking false or less vital targets, the radar surveillance range may be large and only one, or two, aircraft need penetrate to the objective area. Thus, the reconnaissance requirement summarizes the results of this complex analysis, and it is assumed that RED demands a high confidence of meeting this required penetration level.

If the air defense barrier is "tight", meaning that the entire length of the barrier is effectively controlled by a number of stations, then RED's best penetration tactic is to exploit the sectored nature of BLUE's defense by committing all reconnaissance and escort aircraft against the defenses in one sector, in order to saturate the finite detection and intercept capability there. If the CTG plans to use reserved interceptors to back up any sector attacked, then allowance must be made for fighting and saturating such a second wave of interceptors. In this case, RED may also feint attacks in other sectors in order to draw available reserves away from the penetrating group.

However, if the air defense barrier is "loose", meaning that each barrier station can only effectively control some randomly positioned fraction of its assigned sector, then, even though it is assumed that RED cannot locate holes in the barrier, an attack consisting of two, three, or more raid groups with an inter-group spacing that exceeds the effective width of a barrier defense station will succeed (with some probability) in placing two groups in one barrier sector, one of which must penetrate undetected. Thus, loose barriers provide RED with an opportunity to infiltrate one or more groups into the objective area, and a quantitative evaluation of the utility of this tactic is needed.

Thus the looseness of the barrier defines the probability with which various numbers of raid groups, in single or multi-group attacks, will be engaged. The defensive strength of each sector then determines the number of recce aircraft in an intercepted group that will penetrate. The total RED penetration from a multi-group attack is the sum of penetrations from all engaged and non-engaged groups.

If RED does not have the ability to coordinate the search efforts of barrier survivors, then each group in a multi-group attack, by itself, must be strong enough to penetrate sufficient aircraft to meet the reconnaissance requirement. It will be assumed that this is not the case and that search effort by the separate survivors can be coordinated. Since recce aircraft are certain to face further attacks, even after a successful reconnaissance of the objective area, it is necessary that target intelligence data be radioed (directly, or by relay) to the follow-on bomber attack force (or to submarines waiting in the objective area). It is also necessary that RED have a navigation capability sufficiently accurate to permit designation of target locations to the follow-on attack forces.

CTG Response to Barrier Penetration

In addition to the outer air barrier, BLUE may also maintain CAP, or DLI, interceptors in reserve (depending on the proximity of the first barrier to the objective area) to support any sector attacked. Consequently, penetrating groups may face a second attack. If the RED reconnaissance aircraft are known to be unarmed, then BLUE can allow penetrating reconnaissance aircraft (assumed to be electromagnetically silent) to take up mapping positions and then pounce on these when they radiate. This tactic has the advantage that RED cannot feint the reserved interceptors away from genuine penetrating groups, but it does grant RED some opportunity to collect target intelligence before being attacked. Additionally, the tactic works against infiltrating raid groups that were not detected at the barrier. Finally, the CTG must re-configure its defenses upon discovery and prepare to meet a follow-on attack with its reserved AEW/VF units. This defense might consist of a close-in concentrated AEW/CAP defense line protecting the objective area, which corresponds to the assumptions that underlie the earlier air defense model.

SUMMARY

Before proceeding to the development of the CTG air reconnaissance defense model, it is useful to summarize briefly the discussion of the qualitative nature of the defense problem and the key assumptions made.

A CTG is located somewhere in a large objective area with surface units under moderate-to-strict EMCON to reduce the target classification capabilities of enemy reconnaissance aircraft. The CTG may be in a dispersed disposition in an attempt to hide a vital unit formation among several less vital real or decoy formations against enemy airborne radar surveillance. An active air defense barrier, long enough to prevent end runs, is placed outside the objective area across potential attack routes. The barrier is assumed divided into a number of sectors, each of which is provided with a surveillance and intercept capability that may, or may not, be sufficient to guard the full

sector width. RED is assumed to know the center of the objective area, the length of the barrier, and the number and strength of its defense sectors, but not the precise locations of any "holes" in the defense coverage. Using this knowledge, and the capabilities of his reconnaissance sensors, RED specifies a reconnaissance requirement in terms of the number of penetrating reconnaissance aircraft needed to adequately map the objective area. This requirement, plus the selection of a saturation and/or infiltration barrier penetration tactic, implies a minimum-size force of RED reconnaissance aircraft and escort fighters needed to accomplish the reconnaissance mission. RED is assumed to have perfect coordination of surviving reconnaissance aircraft which can radio target intelligence to a follow-on bomber attack force (or a submarine force in the case of the combined air/sub threat). Essentially, all reconnaissance aircraft will be lost, either at the barrier, at reserve defense lines, or in the process of search, so that RED is motivated to minimize the size force committed to reconnaissance, consistent with meeting the mission requirement. BLUE must meet the follow-on bomber attack with reserve interceptors not consumed in the barrier defense, so that BLUE must balance its defense committment between these two closely-spaced threats.

II. STRUCTURE OF THE CTG AIR RECONNAISSANCE ACTIVE DEFENSE METHODOLOGY

INTRODUCTION

The structure of the air reconnaissance active defense methodology consists of three distinct models, linked to one another by the defense functional parameters they share in common. These three models characterize the:

- Barrier geometry, RED penetration tactics, and implied attack engagement probabilities
- Barrier sector fighting strength, and RED's total barrier penetration by engaged and non-engaged raid groups
- Logistics requirements to maintain a barrier of given geometry and strength (without restricting the CV freedom of motion in the objective area).

The purpose of this section is to provide a summary description of the input parameters, structure, and outputs of each model and their interrelations that will help organize the subsequent technical development of each model in Section III.

AIR BARRIER ENGAGEMENT MODEL

Figure 2 depicts three RED reconnaissance raid groups, each consisting of two recce aircraft and three escort fighters, approaching a loose barrier of five stations, each consisting of one AEW and one CAP aircraft. The BLUE aircraft radiate intermittently and move in such a way as to conceal the gaps in the barrier from RED's ELINT capability. The spacing between the RED groups exceeds the coverage of a single station, but not the width of a sector. Consequently, it is possible (not certain) for two raid groups to enter one defense sector. In this case, at least one group (and possibly two) will not be detected by the barrier defense. The assumption that RED knows the density of barrier stations and their coverage is conservative or pessimistic from BLUE's point of view. The object of the Barrier Engagement Model is to convert the input description of the barrier geometry and RED attack structure to an output probability distribution of the number of RED raid groups that will be detected and engaged at the barrier.

Let there be S stations in the barrier, each of which covers a fraction, p, of its assigned barrier sector (see Figure 2, where S = 5). For added generality, assume that if a RED recce group enters a covered portion of the barrier, there is a probability ϵ that the group will be detected and that any escort fighters and CAP interceptors will battle. Furthermore, let ϵ_x denote the probability that interceptors, which either survive or evade the escort defense, will be able to engage the trailing recce aircraft despite enemy maneuver or passive ECM tactics they employ. Assume there are N RED raid groups which are spaced d miles apart, where d is expressed as a fraction of the full sector width so that $1 > d > p$. Then, the Barrier Engagement Model describes the probabilities $E_0, E_1, E_2, \ldots, E_N$ that 0, 1, 2, ..., N of the RED recce groups, respectively, will be engaged at the barrier, as a function of the input parameters S, p, ϵ, ϵ_x, d, and N.

AIR BARRIER PENETRATION MODEL

The function of the Air Barrier Penetration Model is to convert an input description of the fighting strength of RED and BLUE forces to a total RED penetration, given that a definite number of RED recce groups are detected and engaged by sector defenses. The Barrier Engagement Model supplies the probability distribution of the number of such engagements that will take place, as a function of barrier geometry and penetration tactic; while the Barrier Penetration Model supplies the total penetration expected for each number of engagements. The two models taken jointly produce the probability distribution of the expected numbers of recce aircraft that penetrate the air defense barrier.

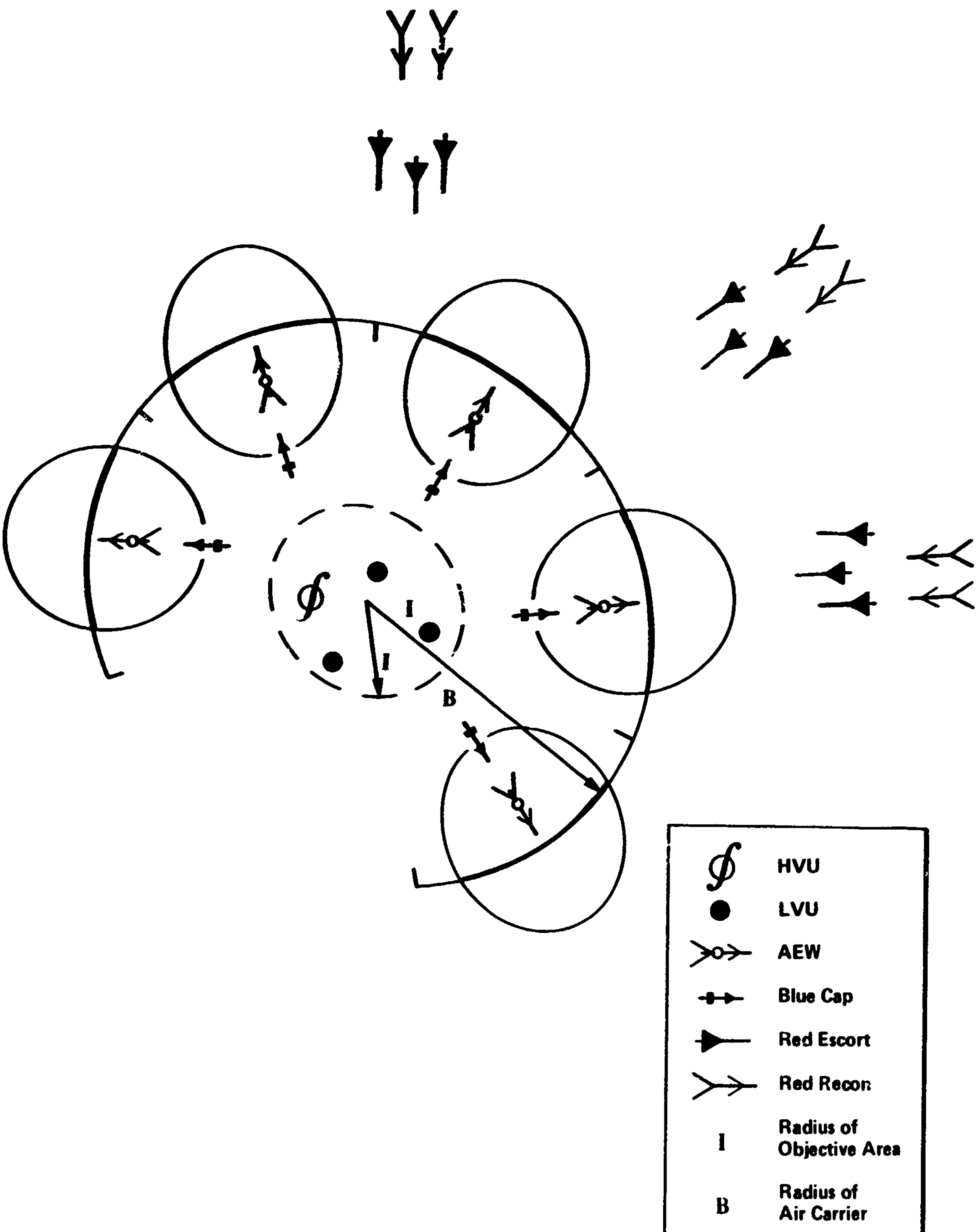


FIGURE 2. BARRIER ENGAGEMENT MODEL: GEOMETRY AND TACTICS

It is admittedly difficult to specify accurate analytic models of the results of two-sided "m against n" air battle engagements. Such a model, nevertheless, is indispensible to further development of the approach. The structure of the analytical methodology proposed in this report, however, is modular, and therefore capable of accepting the output of any air battle model. For present analytical purposes, it is sufficient that the right qualitative structure of air battle attrition be reflected in the model and not necessarily operationally realistic numbers.

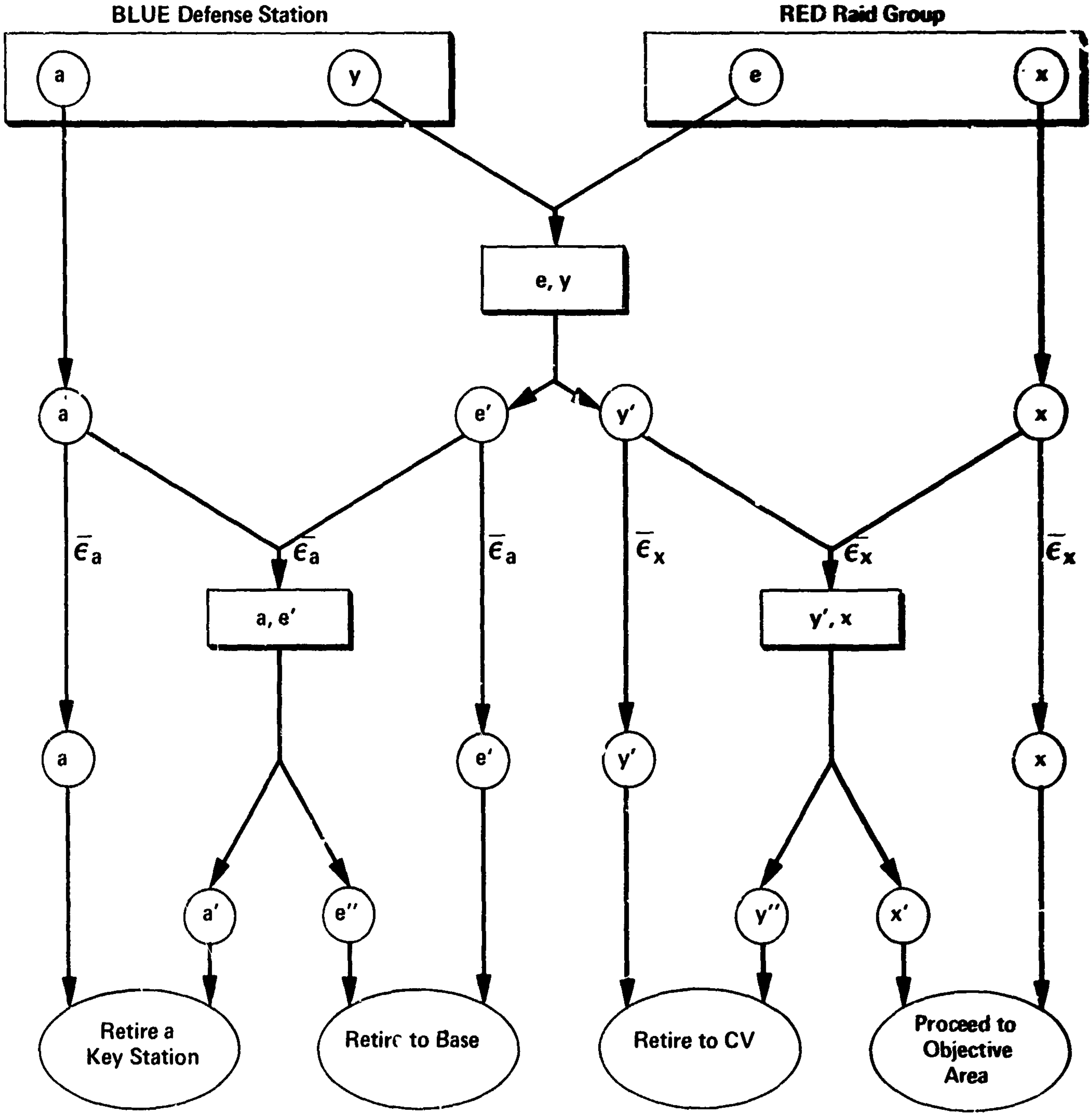
Suppose that an air battle takes place in a barrier sector. Then e RED escort fighters protecting x recce aircraft meet y BLUE interceptors and (for generality) a number, a, of EW aircraft. Figure 3 traces the logic of a hypothetical battle between these forces. The interceptors and fighters meet first and, as a result, y' interceptors and e' fighters survive and proceed to attack the x recce and an AEW aircraft, respectively. With probability ϵ_x, the y' surviving interceptors engage the escorted recce group. As a result, y' exhausted interceptors survive and return to base and x' recce aircraft survive and continue to penetrate. With probability $\bar{\epsilon}_x = 1 - \epsilon_x$, the y' surviving interceptors fail to contact the recce group and return to base. The original x recce aircraft continue their penetration. Symmetrically, with probability ϵ_a, the e' surviving escort fighters contact and fight the AEW aircraft. As a result, e" exhausted fighters survive and return to base and a' AEW survive and maintain their station or return to base. With probability $\bar{\epsilon}_a = 1 - \epsilon_a$, the escort fighters fail to contact the AEW aircraft and return to base and the AEW aircraft maintain their station. Note that the engagement probabilities determine whether or not the recce group is engaged by the defense. The influence of this variable is captured in the Barrier Penetration Model.

For simplicity, assume that any AEW aircraft attacked are lost or driven from position, and concentrate on the defense against recce penetration. Then, a quantitative description of the interceptor/escort, and interceptor/recce battles will complete the analysis. Let $\lambda_b(e,y)$ denote the average probability y'/y that a single interceptor will survive battle with e interceptors. Then, the assumption that the fire power from e fighters is spread evenly across the y interceptors leads to an attrition relationship of "numerically-vulnerable" form

$$\lambda_b(e,y) = \exp(-\beta e/y)$$

where β measures the "efficiency" of a single escort fighter's fire power in the given tactical situation. Consequently, the expected number of surviving interceptors y' out of y is given by

$$y' = y\lambda_b(e,y) = y \exp(-\beta e/y).$$



x RED recce aircraft
e RED escort fighters
y BLUE interceptors
a BLUE PEW aircraft
ϵ probability of joining battle, $\bar{\epsilon} = 1 - \epsilon$
z battle survivors out of z

FIGURE 3. BARRIER SECTOR AIR BATTLE MODEL LOGIC

Assume that the recce aircraft have only a passive defense capability, and that each interceptor has an expected number of L lethal shots (or passes) against a formation of recce aircraft. Then, the expected number of recce survivors x' against a "subtractive" defense is given by

$$x' = (x-y'L)^+$$

where, $(x-y'L)^+ = x - y'L$, or zero, depending on whether or not $x - y'L$ is positive. Consequently,

$$x' = (x-\lambda_b (e,y)\, yL)^+$$

or

$$x' = (x-yL \exp(-\beta e/y))^+ \; .$$

There is a close relationship between the numerically-vulnerable and subtractive attrition models in that a force numerically small relative to the strength of the opposing force is punished severely. This is a qualitatively accurate characteristic of air battles and the basis of the utility of these models in aggregated analytical studies. The parameters a, y, ϵ, x, β, and L measure the defensive strengths of the barrier and the RED force.

To complete the Barrier Penetration Model, it is only necessary to count RED's total penetration from all engaged and non-engaged raid groups. Consequently, suppose that j out of N recce groups are engaged by the defense. If RED commits X recce aircraft and E escort fighters and evenly divides these among N raid groups, then each group contains X/N and E/N aircraft, respectively. If there are y interceptors per barrier station, for a total on-station force of Y = yS interceptors, and each interceptor can destroy L recce aircraft on the average, then the total RED penetration X' from j engaged, and N-j non-engaged, groups is given by

$$X' = (N-j)\, X/N + j(X/N - \lambda_b(E/N,y)yL)^+.$$

RED's combat losses credited to the barrier equal X - X'.

The Barrier Engagement and Penetration Models, in combination, describe the probability distribution of (expected) numbers of penetrating reconnaissance aircraft. For example, suppose that RED uses three raid groups. Then, Table 1 summarizes the probability distribution of expected penetration, where the engagement probabilities E_0, E_1, E_2, and E_3 are functions of the parameters S, p, ϵ, ϵ_x, d, and N = 3, as discussed above.

TABLE 1

PROBABILITY DISTRIBUTION OF EXPECTED NUMBERS OF RECONNAISSANCE AIRCRAFT PENETRATING THE BARRIER

j Engagements	Total Expected Number of Recce Aircraft Penetrating the Barrier, Given j Out of Three Groups Engaged
E_0	X
E_1	$2X/3 + (X/2 - \lambda_b(E/3, y)yL)^+$
E_2	$X/3 + 2(X/3 - \lambda_b(E/3, y)yL)^+$
E_3	$3(X/3 - \lambda_b(E/3, y)yL)^+$

It is useful to note that the functional forms of the Barrier Engagement and Penetration Models are general and not necessarily limited to air defense interpretation. The effectiveness of land, sea, and undersea barriers that can be described by the general functional parameters introduced, can be studied as well. In addition, the conceptual analysis of a barrier into detection, combat, and logistics models is also generally valid.

In order to complete the description of the air reconnaissance active defense effectiveness methodology, it is only necessary to measure the CTG logistics requirements associated with barriers of given extent and combat strength.

AIR BARRIER LOGISTICS MODEL

Up to this point, the geometry and strength of BLUE's barrier has been assumed given. To complete the structure of the air reconnaissance active defense model, it is necessary to characterize the CTG logistics costs associated with a given barrier structure.

Suppose BLUE continuously maintains a barrier station at range B from the CV with an aircraft capable of T hours of flight (allowing for necessary combat and landing fuel reserves) at an economical average cruise speed V. Let H be the deck turn-around time for this aircraft. If the flight time to and from station is not operationally useful, then only T - 2B/V hours on station, out of a total mission/deck-replenishment cycle of T + H hours, are available from one aircraft. Thus, the aircraft duty cycle is at most

$$(T-2B/V)\big/(T+H).$$

Consequently, the number of such aircraft required to continuously man one station equals the reciprocal of the duty cycle, or

$$(T+H)/(T-2B/V).$$

Dividing by T, setting $\tau = H/T$, and noting that the product TV equals the total cruise range, C, of the aircraft, the number of aircraft required to maintain one station equals

$$(1+\tau)/(1-2B/C).$$

Assuming that, on the average, only a fraction, μ, of a pool of such aircraft are maintained in operational condition at any one time, then the total number of aircraft required to continuously man one station equals

$$\frac{(1+\tau)}{\mu}\left(\frac{1}{1-2B/C}\right)$$

a formula that nicely separates the logistics requirements due to maintenance $((1+\tau)/\mu)$ and operational $(1/(1-2B/C))$ factors. Thus, as a first approximation, the total logistics draw, Y_B, to maintain y aircraft on each of S stations, is given by

$$Y_B = y\left(\frac{1+\tau}{\mu}\right)\left(\frac{S}{1-2B/C}\right) .$$

This size force must be increased further to allow the CV full freedom of motion in the objective area while maintaining the barrier from an offset position in the objective area. This logistics increment will be analyzed in Section III.

SUPPLEMENTARY RELATIONS

If BLUE maintains a ring-type barrier that is a fraction F of the full circumference of the circle of range B, then the barrier length equals $2\pi BF$. If y aircraft on station can effectively control a width of barrier of $W_b(y)$ miles, and the fraction of barrier length controlled must equal p, at least, then the number of required barrier stations is given by

$$S = \left\{\frac{2\pi BFp}{W_b(y)}\right\}$$

where the function {z} denotes the next integer larger than (or equal to) the number z. If y refers to interceptor aircraft, then this parameter contributes to the Barrier Engagement Penetration and Logistics Models and represents a common link between all three. The number of sectors, S, is also shared with the Barrier Engagement Model.

Suppose that the barrier station range, B, is chosen so that RED is first-engaged at a range that prevents the recce aircraft from seeing into an operating area of radius I. Let $W_b(y)$ be the diameter of an effective BLUE surveillance disk at the barrier and suppose that a detected RED recce group is engaged at an average distance forward of the barrier of one-half the radius of this disk. Then, the first meeting range $B + W_b(y)/4$ must equal RED's assumed surveillance range $W_r/2$, plus the radius of the operating area I. Consequently,

$$B + W_b(y)/4 = I + W_r/2$$

or

$$B = I + W_r/2 - W_b(y)/4 \ .$$

Thus, all parameters in the active air reconnaissance defense model are related to the operationally significant parameters I, $W_b(y)$, W_r, F, p, ϵ, ϵ_X, d, and N; force sizes X, E, Y_B (=yS); air combat strength β, L; and air logistics quality τ, μ, and C.

STRUCTURE OF THE METHODOLOGY

Table 2 summarizes the structure of the methodology for analyzing the effectiveness of a CTG active defense against air reconnaissance. Each of the three models comprising the methodology can be used, or modified, independently of the others. The three models characterizing the raid detection; interception penetration; and logistics aspects of the total air defense problems are linked through the common values of parameters they share.

Table 3 lists the parameters, and their definitions, that appear in the air reconnaissance active defense effectiveness methodology.

TABLE 2

STRUCTURE OF THE CTG AIR RECONNAISSANCE ACTIVE DEFENSE EFFECTIVENESS METHODOLOGY

AIR BARRIER ENGAGEMENT MODEL

Probability that j of N recce groups are engaged at the barrier as a function of parameters S, $p(y)$, ϵ, ϵ_x, d, and N	E_o	X'	Total expected number of recce aircraft out of X, that penetrate the barrier, given that j out of N groups are engaged, as a function of parameters X, E, y, L, and λ_b
	E_1	$(N-1)\,X/N + (X/N - \lambda_b(E/N, y)yL)^+$	
	⋮	⋮	
	E_j	$(N-j)\,X/N + j(X/N - \lambda_b(E/N, y)yL)^+$	
	⋮	⋮	
	E_N	$N(X/N - \lambda_b(E/N, y)yL)^+$	

AIR BARRIER LOGISTICS MODEL

$Y_B = y\left[((1+\tau)/\mu)\;\;(S/(1-2B/C))\right]$	Total number of BLUE aircraft, of given type, to maintain the barrier as a function of y, S, B, C, τ, μ

SUPPLEMENTARY RELATIONS

$$S = \{2\pi BFp/W_b(y)\}\,;\;\; B = I + W_r/2 - W_b(y)/4;\;\; \lambda_b(e, y) = \exp(-\beta e/y)$$

TABLE 3

DEFINITIONS OF PARAMETERS USED IN THE CTG AIR RECONNAISSANCE ACTIVE DEFENSE EFFECTIVENESS METHODOLOGY

Parameter	Definition
B	Radius to the active air barrier from center of the objective area
I	Radius of circular CTG objective area
F	Fraction of full barrier circumference ($2\pi B$) guarded
S	Number of defense sectors (stations) on the guarded parameter
X	Number of RED recce aircraft committed
E	Number of RED escort fighters committed
y	Number of BLUE interceptors per defense sector
L	Expected number of recce aircraft destroyed per interceptor
N	Number of spaced RED recce groups
d	Separation distance of RED recce groups as a fraction of defense sector width
p	Length of each defense sector covered as a fraction of defense sector width
ϵ	Probability of recce group detection within defense coverage
ϵ_x	Probability of successful interception of recce aircraft, given detection
τ	Deck turn-around time as a fraction of aircraft mission time
μ	Fraction of aircraft in operational condition
C	Aircraft cruise range with combat and landing reserves
$W_b(y)$	Surveillance width of a defense station with y interceptors and 1 AEW
W_r	Surveillance width of recce aircraft (localization of classification threats)
β	Measure of combat effectiveness of RED escord fighters against BLUE interceptors

III. SPECIFICATION AND ANALYSIS OF COMPONENT MODELS

The object of this section is to complete the technical development of each of the three component models introduced in Section II, and develop their quantitative implications.

DEFINITION OF THE AIR BARRIER ENGAGEMENT MODEL

The introductory discussion of the Air Barrier Engagement Model in Section II identified several parameters that characterize the active air barrier geometry and the enemy penetration tactics. These were

- S: the number of defense sectors (stations) in the barrier
- p: the fraction of barrier sector width covered by a defense station
- ϵ: the probability of acquiring and engaging a raid that passes through the sector defense coverage
- ϵ_x: the probability with which the interceptors, that survive the RED escort defense, will acquire and fight trailing recce aircraft
- N: the number of RED recce groups
- d: the spacing between the N groups, as a factor of sector width (where $1 \geq d > p$).

The Air Barrier Engagement Model determines the probabilities $E_0, E_1, \ldots, E_N$ with which 0, 1, 2, ..., N recce groups are engaged by barrier defense stations. The construction of this output probability distribution is described in the following paragraphs for various numbers of recce raid groups.

<u>One Reconnaissance Attack Group</u>. The case of a single reconnaissance group is the easiest. The recce group will surely enter one of the S barrier sectors (no end runs). Since a randomly positioned fraction, p, of the barrier sector is covered by the defense, the raid group will enter this defense coverage with probability p. The group will be acquired, and lead escort elements engaged, with probability ϵ. Any surviving interceptors, as a group, will engage the trailing recce aircraft with probability ϵ_x. Consequently, the probability that the group of recce aircraft will be engaged at the barrier equals the product $\epsilon\epsilon_x$. The probability of no engagement equals $1 - \epsilon\epsilon_x$. Hence

$$E_o = 1 - \epsilon\epsilon_x$$

$$E_1 = p\epsilon\epsilon_x.$$

<u>Two Reconnaissance Attack Groups</u>. Figure 4a illustrates the case of two recce groups approaching a barrier of finite length guarded by $S(\geq 1)$ stations. The point of penetration is (uniformly) randomly selected. In the case shown, the two groups will each, unknowingly, pass through a different sector. The two groups are spaced Δ miles apart. Each sector is f miles wide and the defense station covers W miles of the sector. Assume that RED knows the coverage capability of a defense station (which may be a composite effect from one, or more, AEW/CAP aircraft) under the given tactical conditions (RED and BLUE altitudes, BLUE radar capabilities against these targets in the natural and jamming environment, etc.). It is to RED's advantage that $\Delta > W$. Consequently, assume that $f \geq \Delta > W$. To simplify the later calculations it is convenient to make the weak assumption that the defense covers more than half the sector, i.e., $W > f/2$, which includes all the operationally interesting cases. Thus, $f \geq \Delta > W > \frac{1}{2}$.

With these operationally realistic restrictions, there are only two possible alignments between the attack pattern and the barrier stations:

- Both recce groups enter and overload one sector
- Each recce group enters a different sector.

The probabilities of these alignments are computed as follows.

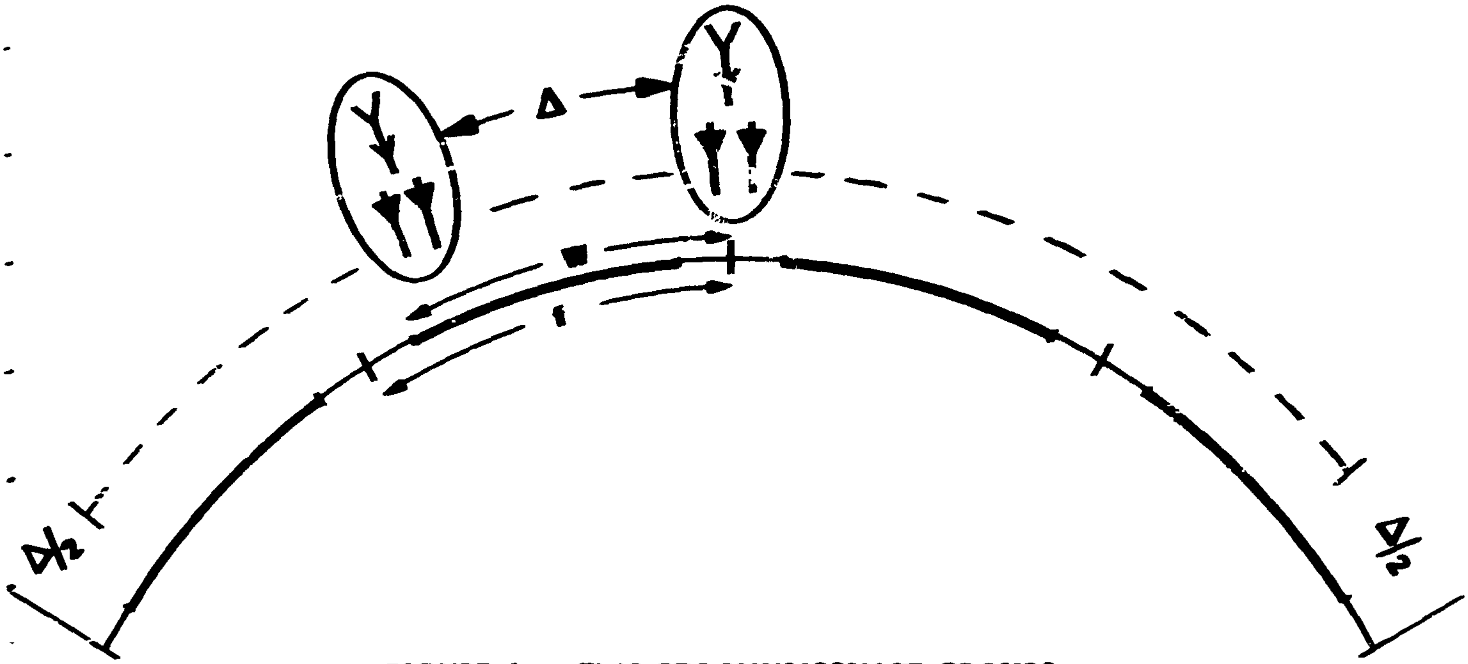


FIGURE 4a. TWO RECONNAISSANCE GROUPS

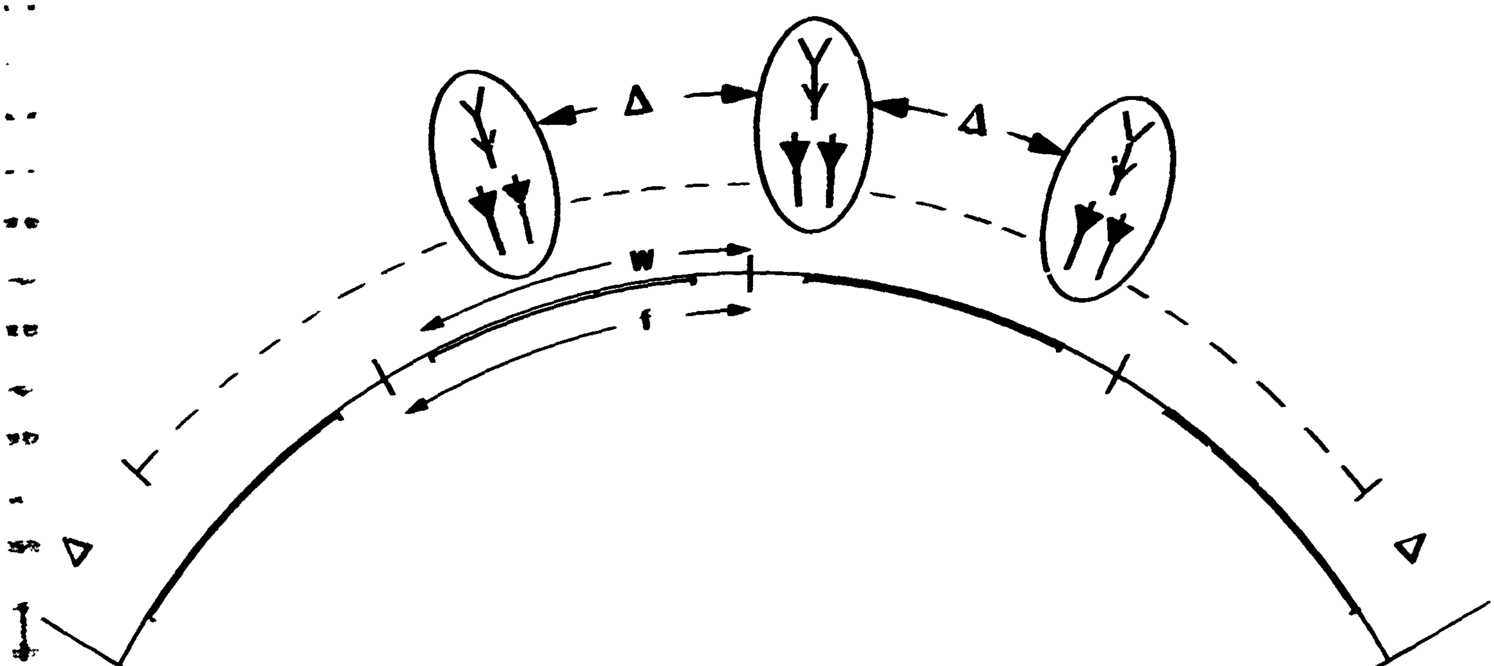


FIGURE 4b. THREE RECONNAISSANCE GROUPS

FIGURE 4. MULTI-GROUP ATTACK/BARRIER-SECTOR ALIGNMENTS

Two groups in one sector: Let A_{20} designate the probability that the two groups enter some one of the S sectors. The two groups must pass through the barrier of total length Sf somewhere (by the assumption that end runs are not possible). Consequently, RED is free to choose the mid-point of his raid pattern uniformly and randomly from a segment of length Sf - 2 $(\Delta/2)$ = Sf - Δ (Thus, there is a slight "edge effect" in the computation for a barrier of finite length.) This "sample space" is shown as a dashed line in Figure 4a. Only a small portion of this sample space of possible mid-points for the raid pattern result in the two groups entering one sector. In fact, in each sector, only a distance of f - Δ miles centered at the mid-point of the sector will permit both raid groups to fit into that sector. Hence, there is a total distance of S(f - Δ) miles out of a possible Sf - Δ that result in a double penetration of one sector. Consequently,

$$A_{20} = S(f-\Delta)/(Sf-\Delta) .$$

Dividing by the sector width f, and recognizing that $\Delta/f = d$,

$$A_{20} = S(1-d)/(S-d).$$

One group in each of two sectors: Since the case of one group in each of two sectors is the only other case possible, the probability, A_{11}, of this attack alignment with the barrier equals $1 - A_{20}$, which, after simplification, is given by

$$A_{11} = \frac{(S-1)d}{S-d} .$$

It is useful to note that in a full ring-type barrier, in which a circle-like enclosed barrier is divided into S sectors, there is no edge effect, and RED can select any point on a circumference of length Sf to be the mid-point of his attack pattern. In this case,

$$A^{o}_{20} = S(f-\Delta)/Sf$$

or

$$A^{o}_{20} = 1 - d,$$

and

$$A^{o}_{11} = d,$$

and the probabilities of the two types of attack alignments are independent of the number of barrier sectors. The same result holds for a barrier with a large number of sectors $(S \to \infty)$ in which the edge effect disappears. (The numerical

results presented in Table 4 show that this effect is small for a wide range of values of S). This last result proves that, in order to maximize the chance of overloading a barrier sector, RED's group spacing must just exceed the width of the defense coverage.

For each attack pattern alignment, it is necessary to compute the probabilities with which 0, 1, or 2 of the recce groups are detected.

Probabilities of Detection In Case A_{11}: Each of the two groups enters a different sector. In each sector, the probability that the group is detected by the defense equals ϵp. Consequently, the joint probability with which 0, 1, or 2 groups are detected is a binominal distribution with two trials and probability of success ϵp. Hence,

$$D_0^1 = (1-\epsilon p)^2$$

$$D_1^1 = 2\epsilon p(1-\epsilon p)$$

$$D_2^1 = (\epsilon p)^2 .$$

This binomial distribution occurs with probability A_{11}.

Probability of Detection In Case A_{20}: Two groups each enter one sector. Since the attack spacing exceeds the width of defense coverage, there is a probability that both groups will escape detection. To determine this probability, note that, wherever the raid pattern lies in the sector, the center of defense coverage can be randomly selected from a segment of length f-W. Out of this segment, there is a distance $\Delta - W$ within which the defense coverage width can fit between the two raid groups. Consequently, the probability that both groups evade the defense coverage equals $(\Delta - W)/(f-W)$ or $(d-p)/(1-p)$. With probability $1-(d-p)/(1-p)$, one of the groups will pass through the defense coverage. The defense could miss this group with probability $(1-\epsilon)$. Hence, the total probability that neither of the groups is acquired equals

$$\frac{d-p}{1-p} + (1-\epsilon)\left(1 - \frac{d-p}{1-p}\right)$$

Simplifying this expression, the probability D_0^2 that neither of the groups is detected is given by

$$D_0^2 = 1 - \epsilon(1-d)/(1-p) .$$

TABLE 4

BARRIER RAID GROUP DETECTION PROBABILITIES: PERFECT DETECTION/INTERCEPTION: OPTIMUM ATTACK GROUP SPACING

		N = 1		N = 2			N = 3			
	P	E_0	E_1	E_0	E_1	E_2	E_0	E_1	E_2	E_3
S = 3	1.0	0	1.0	0	0	1.0	0	0	0	1.0
	.9	.10	.90	.01	.30	.69	.00	.05	.46	.49
	.8	.20	.80	.03	.51	.46	.00	.16	.62	.22
	.7	.30	.70	.05	.65	.30	.01	.24	.64	.09
	.6	.40	.60	.08	.74	.18	.01	.39	.58	.02
S = 6	1.0	0	1.0	0	0	1.0	0	0	0	1.0
	.9	.10	.90	.01	.28	.72	.00	.04	.40	.56
	.8	.20	.80	.03	.48	.49	.01	.14	.57	.28
	.7	.30	.70	.06	.62	.32	.01	.26	.61	.12
	.6	.40	.60	.09	.71	.20	.01	.38	.57	.04
S = ∞ (Ring Barrier)	1.0	0	1.0	0	0	1.0	0	0	0	1.0
	.9	.10	.90	.01	.26	.73	.00	.04	.38	.58
	.8	.20	.80	.03	.46	.51	.00	.14	.55	.31
	.7	.30	.70	.06	.60	.34	.00	.26	.60	.14
	.6	.40	.60	.09	.69	.22	.01	.38	.57	.04

LEGEND:
- N: Number of RED reconnaissance groups
- S: Number of barrier defense sectors
- P: Fraction of a defense sector covered

Since it is impossible that both groups are detected, the probability that one is detected equals $1-D_0^2$. Consequently

$$D_0^2 = 1 - \epsilon(1-d)/(1-p)$$

$$D_1^2 = \epsilon(1-d)/(1-p)$$

$$D_2^2 = 0.$$

This distribution occurs with probability A_{20}.

The total probability D_j, that j groups will be acquired equals the weighted average probability $A_{11}D_j^1 + A_{20}D_j^2$. Consequently,

$$D_0 = A_{11}(1-\epsilon p)^2 + A_{20}\big(1-\epsilon(1-d)/(1-p)\big)$$

$$D_1 = A_{11}\, 2(\epsilon p)(1-\epsilon p) + A_{20}\, \epsilon(1-d)/(1-p)$$

$$D_2 = A_{11}(\epsilon p)^2.$$

Even though the defense acquires the lead elements of a raid it is useful to allow the possibility that the interceptors, after fighting or evading the escort fighter protection, make contact with the recce aircraft only with probability ϵ_x. (The case in which there are no lead raid elements, and the defense directly engages the recce aircraft, is included by setting $\epsilon_x = 1$). Then, the final output probability distribution of 0, 1, or 2 engagements of the recce aircraft is given by

$$E_0 = D_0 + D_1(1-\epsilon_x) + D_2(1-\epsilon_x)^2$$

$$E_1 = D_1\epsilon_x + D_2\, 2\epsilon_x(1-\epsilon_x)$$

$$E_2 = D_2\epsilon_x^2 \quad .$$

The full analytical potential of separating the capabilities for raid detection and the interceptor prosecution are not realized in the model at the present time. In fact, the probabilities ϵ and ϵ_x enter the model as the product $\epsilon\epsilon_x$, which could be represented by a single engagement parameter. However, the defensive consequences of failing to detect and, given detection, failing to prosecute, are markedly different. In the first case, failure to detect means that penetrating groups have slipped passed the barrier without alerting the defense. It is impossible to call reserve forces against such groups. On the other hand, a penetration made because of a failure in prosecution has alerted the defense and reserves can potentially be brought to bear against them. An improved version of the model, that includes reserve defense lines, will incorporate the effects of this distinction.

Three Reconnaissance Attack Groups

The last case to be considered, that of three equally spaced raid groups, is pictured in Figure 4b. Again, the weak and operationally important restriction that $f \geq \Delta > W > \frac{1}{2}$, is made. There are only two possible attack pattern alignments with the barrier sectors:

- Two recce groups enter one sector and a third group an adjacent sector
- Each recce group enters a different sector.

Also note that these restrictions on the geometry imply that $S \geq 2$, since the attack pattern has width 2Δ. Because the method for computing the attack alignment and detection probabilities closely parallels the discussion given for the case $N = 2$, the explanation for this case will be brief.

Each of three groups in a different sector: Let A_{111} denote the probability that each of the three groups enters a different sector. Since RED must pass through the barrier, the position of the middle groups must be chosen from a front of length $SF-2\Delta$, indicated by the dashed line in Figure 4b. In addition, in order to fit three groups into three adjacent sectors, the middle group must be positioned in a segment of length $2\Delta-f$ centered at the mid-point of a sector, that has neighboring sectors to the left and right. Consequently, there is a total distance of $(S-2)(2\Delta-f)$ miles out of $Sf-2\Delta$ miles in which the middle recce group can be placed to yield an alignment of one group to a sector. Hence

$$A_{111} = (S-2)(2\Delta-f)/(Sf-2\Delta),$$ or normalizing distances in terms of sector width,

$$A_{111} = (S-2)(2d-1)/(S-2d).$$

Two groups in one sector, one group in another: Let A_{21} denote the probability of an alignment of two recce groups in one sector, one group in an adjacent sector. Since this is the only other alignment possible, $A_{21} = 1 - A_{111}$. Hence with simplification

$$A_{21} = 2(S-1)(1-d)/(S-2d).$$

In the case of a full ring-type barrier in which a circle-like enclosed barrier is divided into S sectors, there is no edge effect, and RED can select any point on a circumference of length Sf to be the penetration point for his missile recce group. Also, the total length of barrier that produces a (1,1,1) alignment equals $S(2\Delta-f)$. Consequently,

$$A_{111} = 2d - 1$$

$$A_{21} = 2(1-d) \ .$$

As before, RED should minimize his recce group spacing (with the proviso that $d > p$), in order to maximize the probability of overloading a sector.

Probability of Detection In Case A_{111}: Since each of three recce groups is independently detected with probability ϵp, the probability distribution of detecting 0, 1, 2, or 3 recce groups is binomial with three trials, probability of success ϵp. Hence,

$$D_0^1 = (1-\epsilon p)^3$$

$$D_1^1 = 3(\epsilon p)(1-\epsilon p)^2$$

$$D_2^1 = 3(\epsilon p)^2(1-\epsilon p)$$

$$D_3^1 = (\epsilon p)^3 .$$

This distribution occurs with probability A_{111}.

Probability of Detection In Case A_{21}: Using the results from the case of two recce groups, the probability that neither of the two groups in one sector is detected equals $1 - \epsilon(1-d)/(1-p)$. The independent probability that the third group escapes detection too, equals $1 - \epsilon p$, and the joint probability of no detections equals the produce $(1-\epsilon(1-d)/(1-p))(1-\epsilon p)$. The probability of exactly one detection equals the probability that one of two groups in one sector is detected and the third is not, or the single group is detected and the doubled group escapes. Hence the probability of one detection equals

$$\epsilon\left[(1-d)/(1-p)\right](1-\epsilon p) + (\epsilon p)\left(1-\epsilon(1-d)/(1-p)\right) .$$

The probability of two detections equals the product of the probability that the singleton group is detected and the probability that one of the doubled groups is detected. This probability equals $(\epsilon p)\epsilon(1-d)/(1-p)$. The probability of detecting all three groups in this alignment equals zero. Consequently, the probability distribution of recce group detections for alignment pattern (2, 1) is given by

$$D_0^2 = \left[1-\epsilon(1-d)/(1-p)\right](1-\epsilon p)$$

$$D_1^2 = \epsilon(1-d)/(1-p) + \epsilon p(1-\epsilon(1-d)/(1-p))$$

$$D_2^2 = (\epsilon p)\epsilon(1-d)/(1-p)$$

$$D_3^2 = 0 .$$

This probability distribution of detections occurs with probability A_{21}.

The total probability D_j, that j groups will be acquired, equals the weighted average probability $A_{111}D_j^1 + A_{21}D_j^2$. Consequently

$$D_0 = A_{111}(1-\epsilon p)^3 + A_{21}(1-\epsilon(1-d)/(1-p))(1-\epsilon p)$$

$$D_1 = A_{111}3(\epsilon p)(1-\epsilon p)^2 + A_{21}(\epsilon(1-d)/(1-p)/(1-p) + \epsilon p(1-\epsilon(1-d)/(1-p))$$

$$D_2 = A_{111}3(\epsilon p)^2(1-\epsilon p) + A_{21}(\epsilon p)\epsilon(1-d)/(1-p)$$

$$D_3 = A_{111}(\epsilon p)^3.$$

Finally, allowing for the possibility that interceptors fail to contact the recce aircraft followina their battle with RED escort fighters, the probability distribution of the number of recce engagements is given by

$$E_0 = D_0 + D_1(1-\epsilon_X) + D_2(1-\epsilon_X)^2 + D_3(1-\epsilon_X)^3$$

$$E_1 = D_1\epsilon_X + D_2\epsilon_X(1-\epsilon_X) + D_3 3\epsilon_X(1-\epsilon_X)^2$$

$$E_2 = D_2\epsilon_X^2 + D_2\epsilon_X^2(1-\epsilon_X)$$

$$E_3 = D_3\epsilon_X^3 .$$

The same reasoning could be applied to determine the engagement probabilities for four or more equally spaced recce groups, but these three cases are sufficient to reveal the utility to RED of splitting his force in order to increase the probability of penetrating loose barriers.

Implications of the Air Barrier Engagement Model

In order to examine the quantitative implications of the Barrier Engagement Model, when RED divides his reconnaissance force into 1, 2, or 3 groups, the model was programmed, and engagement distributions calculated, for a variety of combinations of barrier geometrics. An important part of these data are

presented in Table 4. In this table, the defense detection and interceptor engagement probabilities, ϵ and ϵ_x, are unity, and RED's inter-group spacing is set at its minimum value (d = p) in order to maximize the probability of overloading a sector. These data reveal the effects of the number of sectors in the barrier, and the looseness of the defense coverage.

Table 4 shows that, while the probabilities of engaging the whole RED force increase with increasing number of sectors, the rise is small over the range from $S = 3$ to $S = \infty$ (the distributions for circular, and finite, barriers are practically indistinguishable, at least beyond 6 stations). Consequently, the engagement probability distributions for the case of a ring barrier are representative of barriers of finite length, too.

Studying the case $S = \infty$, it is clear that RED significantly improves his chances of infiltrating at least one group through a loose barrier by dividing his force. For example, when BLUE covers 80% (p = 0.8) of the full ring barrier, there is a .20 chance that a single recce group will penetrate; a .49 chance that at least one of two groups will penetrate; and a .69 chance that at least one of three groups will penetrate. The corresponding figures for a 70% (p = 0.7) barrier coverage are .30, .66, and .86. Consequently, BLUE cannot allow the barrier coverage to fall much below complete coverage without inviting RED to space his attack to infiltrate the barrier. However, even at 70% coverage, if RED uses 3 recce groups, there is still a .14 probability that RED's entire force will be engaged, and on highly unfavorable terms, since each group (which is only a third the size of the total force input) will have to take on the defenses in a sector. Thus, RED loses the important effect of mass saturation in a sector in order to exploit an opportunity to infiltrate past the barrier defense. (The consequences of this will be taken up in the following discussion of the Barrier Penetration Model. It should be emphasized that these data do not show the probability distribution of the <u>numbers</u> of aircraft that penetrate.) In general, it is clear that when RED's entire available force is weak, compared to the defensive strength of a sector, but BLUE's coverage is loose, RED's best (or only) chance to penetrate is to divide his force and hope to infiltrate unseen past the defense. If his force is strong relative to the strength of a sector and any reserve forces (not feinted) that can be brought to bear, RED is motivated to mass in one group. In between these polar cases, it is not obvious how RED would weigh the prospects of fighting versus infiltrating the barrier. The fewer barrier stations BLUE uses, then the stronger each can be, for a given force committment, but the barrier coverage is reduced inviting RED to infiltrate. On the other hand, if BLUE extends his coverage, the defensive strength of each station suffers inviting RED to mass and saturate the fighting strength in a sector. In any case, BLUE's coverage must be held fairly high, so the difference to BLUE may not be great (this is borne out by data in the discussion of the Barrier Logistics Model).

Table 5 expands the engagement probability data for the case of a ring barrier to include the effects of imperfect detection and interceptor engagement capabilities (ϵ and ϵ_x less than 1). Again, these data show a strong increase in the probability that at least one group will penetrate, with division of the RED force. However, the incentive for RED to divide his force and try to infiltrate is even greater here because (even though the probability of penetration of a single group is higher with leaky defenses) the probability that all the groups of a multi-group attack will be engaged is very low. For example, in the case of 80% coverage with perfect detection and interception, a single group will penetrate with probability .20 and at least one of three-groups with probability .69. If the detection and interception probabilities each fall to 0.8, then single group penetration rises to .49, still a risky proposition, while the probability that at least one group of a three-group attack will penetrate equals .92. Thus, somewhat loose coverage, plus imperfect detection or interception capabilities, invites infiltration tactics (with the caution that if the number of aircraft required to penetrate is large, infiltration may not be able to deliver this number with high confidence because each raid group is necessarily small).

The data in Table 5 show the complete symmetry of the effects of ϵ and ϵ_x. In the model as presently constructed, the detection and interceptor contact probabilities enter simply as the product $\epsilon\epsilon_x$. However, the potential significance of the separate treatment of detection and interceptor prosecution capabilities can be seen as follows. For the case of 100% barrier coverage, detection probability 0.6, and interception probability 1.0, the probability that at least one of three raid groups penetrates equals .78. However, because interception, given detection, is perfect, these penetrating groups must have passed the barrier undetected by the defense. They have infiltrated without alerting the defense, so there is no chance to call reserves to meet them. However, if the detection probability is perfect, and the interception probability 0.6, the probability of at least one group penetrating still equals .78, but some number of these groups had been detected, but interceptor contact was missed. Nevertheless, reserves called to the sector may be able to acquire these recce groups. This aspect of the defense will be analyzed in a later report, along with the model for management of reserved interceptors.

The penetration probabilities shown in Tables 4 and 5 assume that RED knows the defense coverage width exactly. By setting his spacing slightly larger than the width of defense coverage, RED maximizes the probability of penetrating at least one group of a multi-group attack. However, if RED spacing can not be so finely controlled, RED penetration will decrease for under, or overestimates of the defense coverage width, or mis-estimates of the number of stations guarding the barrier. The effects on the riskiness of spaced attacks to RED will also be explored with an improved version of the Barrier Engagement Model. Preliminary results show that at high levels of coverage (75%-100%), the effect of RED overestimating the defense station coverage results in a modest degradation in penetration effectiveness.

TABLE 5

BARRIER ENGAGEMENT PROBABILITIES: IMPERFECT DETECTION (ϵ<1) AND INTERCEPTION (ϵ_x<1); OPTIMUM ATTACK GROUP SPACING; RING-TYPE BARRIER

			N = 1		N = 2			N = 3			
P	ϵ	ϵ_x	ϵ_0	ϵ_1	ϵ_0	ϵ_1	ϵ_2	ϵ_0	ϵ_1	ϵ_2	ϵ_3
1.0	1.0	1.0	0	1.0	0	0	1.0	0	0	0	1.0
		.8	.20	.80	.04	.32	.64	.01	.10	.38	.51
		.6	.40	.60	.16	.48	.36	.06	.29	.43	.22
	.8	1.0	.20	.80	.04	.32	.64	.01	.10	.38	.51
		.8	.36	.64	.13	.46	.41	.05	.25	.44	.26
		.6	.52	.48	.27	.50	.23	.14	.39	.36	.11
	.6	1.0	.40	.60	.16	.48	.36	.06	.29	.43	.22
		.8	.52	.48	.27	.50	.23	.14	.39	.36	.11
		.6	.64	.32	.41	.46	.13	.26	.44	.25	.05
.8	1.0	1.0	.20	.80	.03	.46	.51	.00	.14	.55	.31
		.8	.36	.64	.14	.53	.33	.06	.31	.47	.16
		.6	.52	.48	.30	.52	.18	.17	.43	.33	.07
	.8	1.0	.36	.64	.14	.53	.33	.06	.31	.47	.16
		.8	.49	.51	.26	.53	.21	.14	.42	.36	.08
		.6	.62	.38	.41	.47	.12	.27	.46	.24	.03
	.6	1.0	.52	.48	.30	.52	.18	.17	.43	.83	.07
		.8	.62	.38	.41	.47	.12	.27	.46	.24	.03
		.6	.71	.29	.53	.40	.07	.40	.44	.15	.01

LEGEND:
P: Fraction of barrier covered
ϵ: Probability of raid detection in each sector
ϵ_x: Probability of recce interception, given raid detection
N: Number of RED reconnaissance groups

DEFINITION OF THE AIR BARRIER PENETRATION MODEL

Resume

The Air Barrier Engagement Model describes the probability distribution of the number of recce groups undetected (engaged) in a spaced, multi-group attack pattern. The Air Barrier Penetration Model describes the results of these engagements by calculating the total number of recce aircraft that penetrate the barrier, either by fighting the sector defenses, or infiltration past these. Thus, a description of the "fighting strength" of each barrier station against a recce group that may be protected by escort fighters is required.

Figure 3 described an assumed logic for the air battle in a single defense sector. In this battle logic e RED escort fighters attack y BLUE interceptors, and a AEW aircraft in order to reduce the threat to x trailing recce aircraft. Let $\lambda_b(e,y)$ denote the probability of survival of a single interceptor in this air battle. Then, an expected number of interceptors $y' = \lambda_b(e,y)y$ will survive, or evade, the RED attack and proceed to acquire and fight the trailing recce aircraft. If the y' surviving interceptors acquire the recce aircraft (probability ϵ_x), and if each interceptor can be expected to kill L of these, then x', the total number of recce aircraft that survive the engagement, is given by

$$x' = \left(x - \lambda_b(e,y)yL\right)^+$$

(where $(a-b)^+ = (a-b)$, or zero, depending on whether or not (a-b) is positive). The key feature of such a "subtractive defense" is that if RED's commitment, x, is below a certain threshold, there is no penetration. Once this threshold commitment is met, penetration increases one-for-one with the size of the commitment. Thus, the BLUE fighting strength is assumed to be saturable. RED can control the size of this threshold by attacking and destroying defensive interceptors sent to meet the recce aircraft. Hence, RED can substitute escort fighters for recce aircraft while maintaining a given level of penetration. However, because of the short radius of action of many fighter types, even with re-fueling, escort protection may not always be feasible. In this case, RED can penetrate by infiltration and/or by massing recce aircraft against a saturable defense sector, but losses will be higher.

If RED presents a total of X recce and E escort aircraft equally divided among N suitably spaced raid groups, and if the barrier engages j of these groups, then the total number, X', of recce aircraft that will penetrate the barrier is given by

$$X' = (N-j)(X/N) + j(X/N - \lambda_b(E/N,y)yL)^+.$$

The implications of this type of Air Battle Penetration Model are discussed next.

Implications of the Air Barrier Penetration Model

Table 6 lists the total RED penetration for various RED force sizes, numbers of equal-size raid groups, and BLUE sector defense strength, as a function of the number of raid groups engaged by the air barrier. This case assumes that RED has no escort fighter protection, so that $\lambda_b(e,y) = 1$. Typical probability distributions for the numbers of engagements are given in Tables 4 and 5.

In the case of one raid group, Table 6 shows (case N = 1) the characteristic threshold effect of a subtractive defense with one-for-one penetration with increasing force size beyond a given threshold. Regardless of the fraction of barrier covered and the defensive strength in each sector, RED can penetrate by using a large enough force. However, a loose barrier coverage provides an opportunity for RED to minimize his force commitment. If RED must deliver one aircraft to the CTG objective area, against sectors of strength yL = 3, yL = 6, and yL = 9, RED needs 4, 7, and 10 aircraft respectively in a single-group raid. But, an attack by only three aircraft one in each of three spaced groups, threatens to infiltrate at least one aircraft with some probability, regardless of the sector defense strength. For example, if the coverage in a ring-type barrier is 80%, the data in Table 4 shows that RED has a .69 probability of delivering at least one aircraft by this tactic. The attractiveness of the infiltration tactic also increases markedly with the defense strength of each sector, since four aircraft are needed for a saturation penetration when yL = 3, but 10 when yL = 9.

However, as RED's delivery requirement increases, there is less incentive for RED to infiltrate. For the minimum-size of each raid group rises and there is a multiplicative increase in required force size. For example, five aircraft in one raid group deliver two aircraft with certainty in a saturation attack when yL = 3, but six aircraft are required in a three-group raid to deliver at least two aircraft with a probability less than one.

Thus, tight barrier coverage, weak sector defensive strength, and a penetration requirement that exceeds one or two aircraft, motivate RED to make massed, saturation-type, attacks. Loose barrier coverage, strong sector defenses, and low delivery requirements motivate infiltration-type attacks.

If RED can attack, or ward off, interceptor attacks on the recce aircraft, BLUE's subtractive defense threshold (yL) is lowered so that fewer recce aircraft are required for a given level of penetration. Table 7 illustrates this effect by tabulating RED penetration in a single sector, for various escort and recce aircraft raid group compositions against 1, 2, and 3 BLUE interceptors each capable of 3 lethal shots against recce aircraft. BLUE survival probability is assumed to be of numerically-vulnerable form

TABLE 6

TOTAL PENETRATION OF RECONNAISSANCE AIRCRAFT BY FORCE SIZE, PENETRATION TACTIC, AND BARRIER STRENGTH

X Number of RED Recce Aircraft

X	N = 1		N = 2			N = 3			
	E_0	E_1	E_0	E_1	E_2	E_0	E_1	E_2	E_3
1	1	0							
2	2	0	2	1	0				
3	3	0				3	2	1	0
4	4	1	4	2	0				
5	5	2							
6	6	3	6	3	0	6	4	2	0

BLUE Sector Defense Strength, yL = 3

1	1	0							
2	2	0	2	1	0				
3	3	0				3	2	1	0
4	4	0	4	2	0				
5	5	0							
6	6	0	6	3	0	6	4	2	0
7	7	1							
8	8	2	8	4	0				
9	9	3				9	6	3	0

BLUE Sector Defense Strength, yL = 6

1	1	0							
2	2	0	2	1	0				
3	3	0				3	2	1	0
4	4	0	4	2	0				
5	5	0							
6	6	0	5	3	0	6	4	2	0
7	7	0							
8	8	0	8	4	0				
9	9	0				9	5	3	0
10	10	1	10	5	0				
11	11	2							
12	12	3	12	6	0	12	6	4	0

BLUE Sector Defense Strength, yL = 9

N: Number of RED Raid Groups

TABLE 7

SINGLE-SECTOR PENETRATION BY ESCORTED RECONNAISSANCE AIRCRAFT

e: RED Escort Fighters

X RED Recce Aircraft	0	1	2	3
1	0	0	.3	.6
2	0	.5	1.3	1.6
3	0	1.5	2.3	2.6
4	1	2.5	3.3	3.6
5	2	3.5	4.3	4.6
6	3	4.5	5.3	5.6

Number of BLUE Interceptors, y = 1 (yL = 3)

e: RED Escort Fighters

X	0	1	2	3	4	5	6
1	0	0	0	0	0	0	.3
2	0	0	0	0	.5	.9	1.3
3	0	0	0	.9	1.5	1.9	2.3
4	0	0	1	1.9	2.5	2.9	3.3
5	0	.8	2	2.9	3.5	3.9	4.3
6	0	1.8	3	3.9	4.5	5.3	5.3
7	1	2.8	4	4.9	5.5	5.9	6.3
8	2	3.8	5	5.9	6.5	6.9	7.3
9	3	4.8	6	6.9	7.5	7.9	8.3

Number of BLUE Interceptors, y = 2 (yL = 6)

e: RED Escort Fighters

X	0	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	.3	.6	.9
3	0	0	0	0	0	.2	.8	1.3	1.6	1.9
4	0	0	0	0	.4	1.2	1.8	2.3	2.6	2.9
5	0	0	0	.5	1.4	2.2	2.8	3.3	3.6	3.9
6	0	0	.3	1.5	2.4	3.2	3.8	4.3	4.6	4.9
7	0	0	1.3	2.5	3.4	4.2	4.8	5.3	5.6	5.9
8	0	.9	2.3	3.5	4.4	5.2	5.8	6.3	6.6	6.9
9	0	1.9	3.3	4.5	5.4	6.2	6.8	7.3	7.6	7.9
10	1	2.9	4.3	5.5	6.4	7.2	7.8	8.3	8.6	8.9
11	2	3.9	5.3	6.5	7.4	8.2	8.8	9.3	9.6	9.9
12	3	4.9	6.3	7.5	8.4	9.2	9.8	10.3	10.6	10.9

Number of BLUE Interceptors, y = 3 (yL = 9)

Data for: $x^1 = \left(x - yL\exp(-\beta e/y)\right)^+$

$L = 3$, $\exp(-\beta) = 0.5$

$$\lambda_b(e,y) = \exp(-\beta e/y)$$

where the defense "efficiency factor", β, is chosen so that $\lambda_b = .50$ for battles between matched forces $e = y(\beta = .693)$.

The data in Table 7 shows that, for a given required level of penetration, a high rate of substitution of recce aircraft for fighters is available up to the point where the number of escort fighters equals the number of interceptors. Beyond this point, the rate of substitution is significantly lower. For example, for a required penetration of one recce aircraft, the rates of substitution of recce aircraft for fighters equals 1/1, 1.5/1 and 1.5/1 over the range $0 \leq e \leq y$ for $y = 1$, 2, and 3 interceptors, respectively. The corresponding rates of substitution over the range $y \leq e \leq 3y$ for $y = 1$, 2, and 3 are (approximately) .5/1, .5/1, and .5/1 respectively. Thus, if one recce aircraft is at least as valuable as one escort fighter to RED, and it is feasible for RED to escort the reconnaissance raid, then there is a strong incentive for RED to provide somewhat more fighters per raid group than there are interceptors per barrier sector (assuming 50% attrition factor). Beyond this matching point, increased penetration is best achieved by adding recce aircraft to the raid group. The optimum fighter complement does depend upon the combat efficiency of the fighters against the BLUE interceptors and their battle tactics, but the qualitative feature remains that, beyond a certain number of fighters, RED penetration is best increased by adding recce aircraft.

DEFINITION OF THE AIR BARRIER LOGISTICS MODEL

Resume

The Barrier Engagement and Penetration Models characterize RED's expected penetration against barriers of given geometry, detection efficiency, and relative combat strength. The Barrier Logistics Model completes the air reconnaissance active defense effectiveness methodology by characterizing the numbers of aircraft of given capability that must be supplied to maintain barriers of given range, coverage, and strength.

It is also necessary that the CVA(s) be able to maintain the outer active defense barrier from any position within a large objective area (see Figures 1 and 2) in order to prevent RED from inferring the location of vital CTG units by examining the barrier geometry. The characterization of the incremental increase in logistics backup imposed by the requirement of unrestricted CVA mobility in a large objective area is the primary task of this section.

The fundamental formula for the total number, Y_B, of aircraft required to maintain y aircraft on each of S barrier stations, derived in Section II, is given by

$$Y_B = y\left(\frac{1+\tau}{\mu}\right)\left(\frac{S}{1 - 2B/C}\right)$$

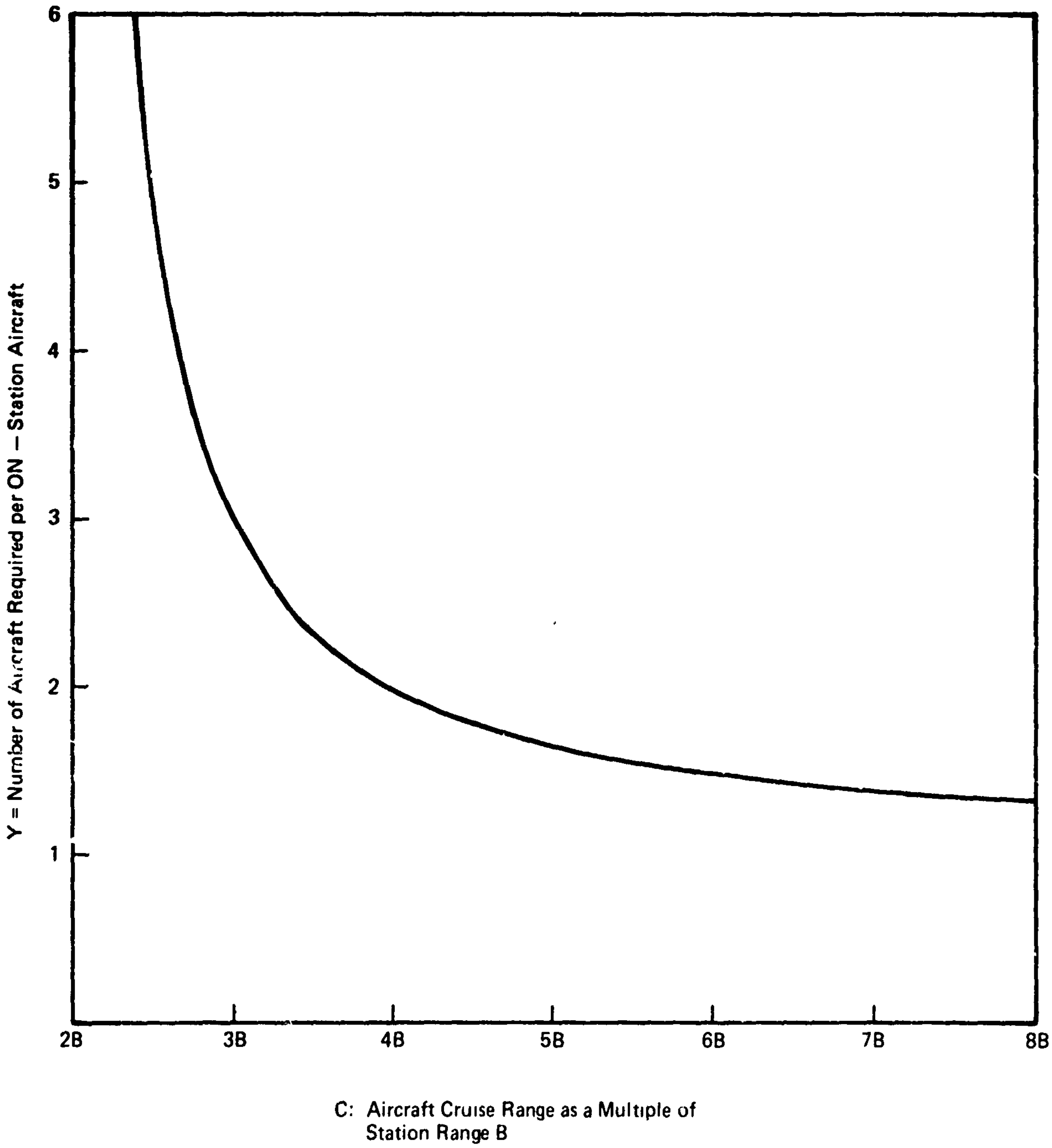
where τ denotes the deck turn-around time of the aircraft, as a ratio of mission flight time; μ, the availability cf the aircraft; C, the mission range (allowing for combat and landing fuel reserves); and B, the station range from the center of the objective area. Thus, the effects of aircraft maintainability and operational employment, on the logistic draw, can be studied separately.

The Operational Employment Factor

The operational employment factor $1/(1-2B/C)$ specifies the minimum number of aircraft required to continuously patrol a barrier station at range B from the CV under the strict assumption that flight time to and from the barrier has no operational value. For a given station range, the larger the aircraft cruise range C, the smaller the logistics draw, since time-on-station becomes a larger fraction of total mission time. Figure 5 graphically shows the highly non-linear effect of aircraft cruise range (as a multiple of station range B) on the logistics draw. If the aircraft cruise range falls below three times the station range ($C < 3B$), the logistics draw due to this factor rises very sharply from a multiple of 3/1. On the other hand, the incremental reduction in logistics draw from extended cruise ranges (as a multiple of station range) is quite modest beyond $C = 4B$, falling to 1.33/1 at $C = 8B$ from 2/1 at $C = 4B$. Thus, the practically significant aircraft cruise ranges would appear to lie between three and six times the station range. These ranges correspond to a logistics draw between 3/1 and 1.5/1. Between four and six times the station range, the change in logistics draw is small. This fact is important to the analysis of the incremental effect on total logistics draw caused by maintaining a barrier from a CV position offset from the center of the objective area.

Station-Keeping From Offset CV Positions

Figure 6 illustrates the problem of maintaining stations on a ring barrier from a CV position offset from the center of the barrier. Here, the CTG is maintaining six stations on a ring of radius B miles. The station legs nearest the barrier are shorter than B miles, while the station legs in the opposite direction are longer than B miles. Thus, there are opposing influences on the total logistics draw. However, because of the highly non-linear effect of station range on logistics draw, the logistics penalty suffered on the longer legs may exceed the gain on the shorter legs. This will be the case where the aircraft cruise range is less than about four times the station range B, placing the problem in the rapidly increasing portion of the logistics draw function (Figure 5). However, when the cruise range exceeds 4B, the two opposing effects should cancel, approximately, and a significant degree of mobility in a reasonably large operating area should be available.



Y = 1/61 – 2B/C

FIGURE 5. LOGISTICS DRAW DUE TO STATION RANGE

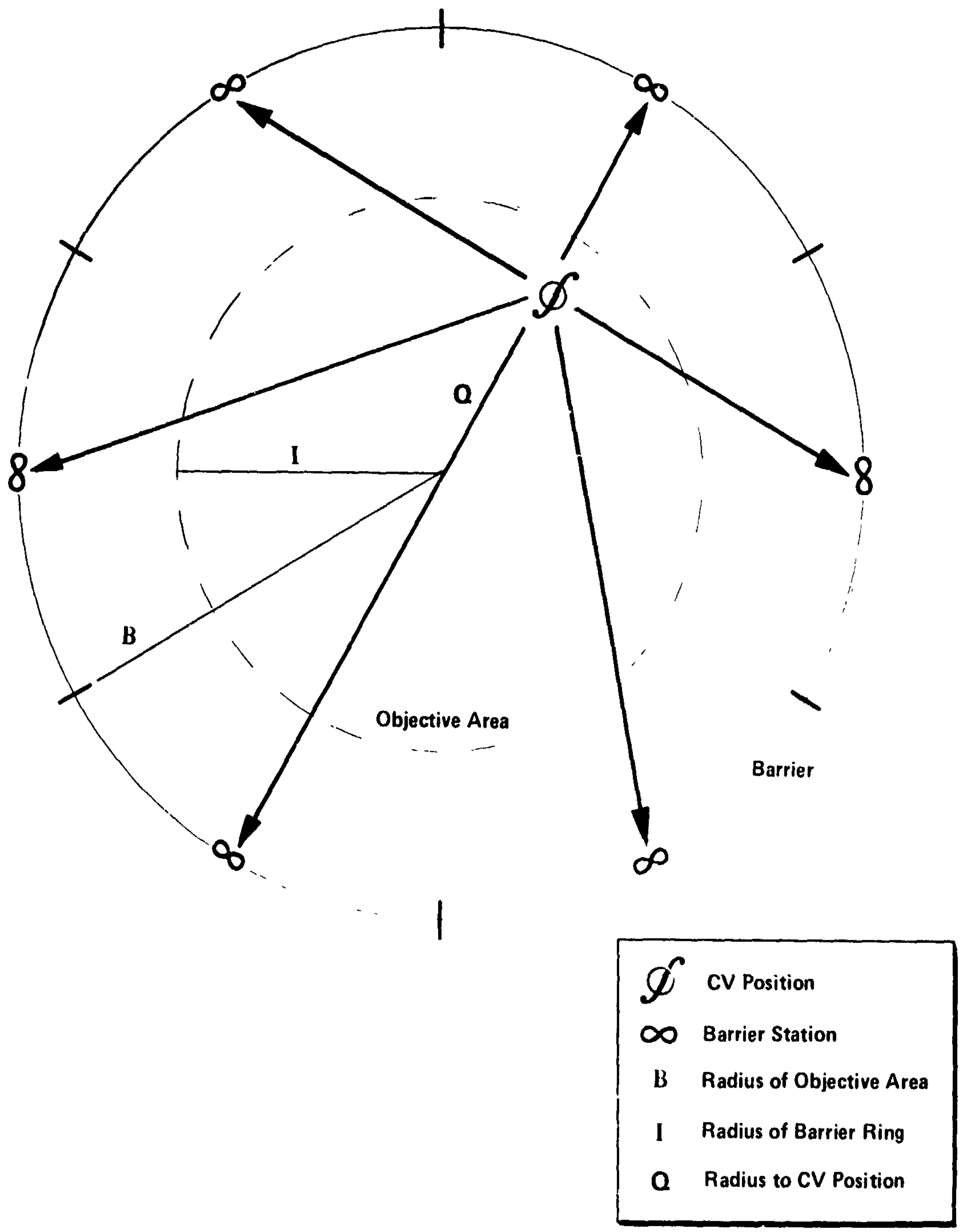


FIGURE 6. AIR BARRIER STATION-KEEPING FROM OFFSET CV POSITION

In order to test the effects of station-keeping from an offset position, a direct computation of the logistics draw associated with maintaining various numbers of stations at several ranges from various positions in an operating area was made. The results are essentially independent of the stationing pattern around the CV, but patterns were chosen that included a longest possible leg, thus maximizing the logistics draw due to station range. No allowance for maintanance and deck turn-around factors are included at this point.

Some results of these calculations are shown in Table 8 for a range of representative situations. The first column of Table 8 displays the total logistics draw required to maintain various numbers of stations at ranges 200, 300, and 400 miles from the barrier center, from several offset radii, when the aircraft cruise range C equals three times the barrier range. When C = 3B, the base logistics draw equals 3S when the CV is at the center of the barrier. These data show that for B = 200, there is a modest increase in the base draw, 3S, for offset distances up to about 50 miles (2, 3, or 4 aircraft for S = 2, 6, 10) and a sharper increase thereafter. This is due to the short combat range (600 nmi) of the aircraft relative to the longest possible station leg (600 nmi). When B = 300 or 400 nmi, the aircraft cruise range C = 3B (900 and 1200 nmi) increases relative to the longest legs (400 and 500 nmi) and there is a smaller increase in logistics draw with offset distance. In these cases, an additional two or three aircraft suffice to give free mobility out to at least 80 miles from the barrier center, whereas this number only gives a free mobility out to 50 miles when B = 200.

The situation changes markedly when aircraft range equals four or six times the barrier range, as shown in the second and third columns of Table 7. Here, 1, 2, or 3 additional aircraft (S = 2, 6, 10) give free mobility out of 80 nmi when B = 200, C = 4B, while one additional aircraft suffices when B = 200, C = 6B, for up to 10 barrier stations. When B = 300 or 400 and C = 4B or 6B, 0 or 1 additional aircraft over the base draw provides CV mobility beyond 100 miles from the barrier center.

Thus, the increase in logistics draw over the base requirement S/(1-2B/C) can be approximated by adding 0, 1, 2, 3, or 4 aircraft to this requirement depending on the values of S, B, and C. (However, when B = 200 and C = 3B, a free mobility only out to 50 miles, and not 100 miles, can be had, otherwise, free CV mobility out to about 100 miles and beyond is available.)

Let $\{a\}$ denote the next integer larger than (or equal to) the number a. Then, the logistics draw (due only to the operational factor) associated with the continuous maintenance of S stations on a circle of radius B with an aircraft of C miles range, free mobility of about 100 miles, is given by

$$\{X/(1-2B/C)\} + y_I$$

TABLE 8

NUMBER OF AIRCRAFT OF CRUISE RANGE "C" REQUIRED TO PATROL "S" STATIONS ON A CIRCLE OF RADIUS "B" FROM A POSITION "Q" MILES FROM BARRIER CENTER (MAINTENANCE FACTORS EXCLUDED)

Q: Distance from Barrier Center

S: Number of Barrier Stations

B = 200 C = 600 3

S	0	20	40	60	80	100
2	6.0	6.2	7.1	9.4	16.7	*
4	12.0	12.3	13.4	16.0	23.6	*
6	18.0	18.5	20.0	23.6	33.4	*
8	24.0	24.6	26.7	32.5	44.0	*
10	30.0	30.8	33.4	39.3	54.8	*

B = 200 C = 800 2

S	0	20	40	60	80	100
2	4.0	4.0	4.2	4.4	4.8	5.3
4	8.0	8.1	8.2	8.6	9.1	9.9
6	12.0	12.1	12.4	12.9	13.6	14.8
8	16.0	16.1	16.5	17.2	18.2	19.7
10	20.0	20.2	20.6	21.4	22.7	24.6

B = 200 C = 1200 1

S	0	20	40	60	80	100
2	3.0	3.0	3.0	3.1	3.1	3.2
4	6.0	6.0	6.1	6.1	6.2	6.4
6	9.0	9.0	9.1	9.2	9.4	9.6
8	12.0	12.0	12.1	12.3	12.5	12.8
10	15.0	15.0	15.2	15.3	15.6	16.0

B = 300 C = 900 2

S	0	20	40	60	80	100
2	6.0	6.1	6.5	7.1	8.4	10.8
4	12.0	12.1	12.6	13.5	14.8	17.5
6	18.0	18.2	18.8	20.0	22.1	25.7
8	24.0	24.3	25.1	26.[illegible]	29.4	34.2
10	[illegible]	30.3	31.4	33.4	36.9	42.7

B = 300 C = 1200 1

S	0	20	40	60	80	100
2	4.0	4.0	4.1	4.2	4.3	4.5
4	8.0	8.0	8.1	8.2	8.5	8.7
6	12.0	12.0	12.2	12.4	12.7	13.1
8	16.0	16.1	16.2	16.[illegible]	16.9	17.5
10	20.0	20.1	20.3	20.6	21.1	21.8

B = 300 C = 1800 0

S	0	20	40	60	80	100
2	3.0	3.0	3.0	3.0	3.1	3.1
4	6.0	6.0	6.0	6.1	6.1	6.2
6	9.0	9.0	9.0	9.1	9.2	9.3
8	12.0	12.0	12.1	12.1	12.2	12.3
10	15.0	15.0	15.1	15.2	15.3	15.4

B = 400 C = 1200 1

S	0	20	40	60	80	100
2	6.0	6.1	6.2	6.6	7.1	8.0
4	12.[illegible]	12.1	12.3	12.7	13.4	14.4
6	18.0	18.1	18.5	19.1	20.7	21.5
8	24.0	24.2	24.6	25.4	26.7	28.6
10	30.0	30.2	30.8	31.6	33.4	35.9

B = 400 C = 1600 1

S	0	20	40	60	80	100
2	4.0	4.0	4.0	4.1	4.2	4.3
4	8.0	8.0	8.1	8.1	8.2	8.4
6	12.0	12.0	12.1	12.2	12.4	12.6
8	16.0	16.0	16.1	16.3	16.5	16.9
10	20.0	20.0	20.2	20.3	20.6	21.0

B = 400 C = 2400 0

S	0	20	40	60	80	100
2	3.0	3.0	3.0	3.0	3.0	3.0
4	6.0	6.0	6.0	6.0	6.1	6.1
6	9.0	9.0	9.0	9.1	9.1	9.1
8	12.0	12.0	12.0	12.1	12.1	12.2
10	15.0	15.0	15.0	15.1	15.2	15.2

C = 3B C = 4B C = 6B

* Longest station leg leaves no time-on-station.

where y_I = 0, 1, 2, 3, or 4 depending upon the parameter values S, B, and C, as shown in Table 7.

While the effect of maintaining the air defense barrier with two (or more) CV has not been studied, the relative insensitivity of the logistics draw with offset CV position, when aircraft cruise range exceeds four times the longest station leg, suggests that there will be no significant reduction in logistics requirements in the case of multiple CVs. However, when aircraft range is less than four times barrier range, two or more CVs are needed to make the barrier feasible.

The Maintainability Factor

The effect on logistics draw due to the maintainability factors τ and μ is shown in Figure 7, which displays the combinations of τ and μ that result in given logistics ratios $(1+\tau)/\mu$. If the boxed region of the (τ, μ) diagram encompassing availability fractions between 0.5 and 0.7, and deck turn-around fractions between 0.1 and 0.5, approximates ranges of practical interest, then the logistics draw due to maintainability considerations rises from a low of 1.5/1 to a high of 3/1 across the boxed region, with most of the relevant region falling somewhat below 2/1 and somewhat above 2.5/1. It is interesting to note that the logistics draws due to station range and maintainability are of equal relative significance for usual station ranges. Thus, improvements in range extension or maintainability provide comparable reductions in logistics requirements.

The most questionable assumption in the logistics formula is that there is a steady-state fraction μ of aircraft in operating condition at any one time. Because of the limited number of maintenance stations on a CV, the rate of arrival of aircraft failures to these stations during intensive operations is likely to exceed their capacity, and maintenance queues will build up. Thus, the typical pattern will be a steady decline in numbers of available operationally useful aircraft during an on-line period. Thus, two, or more, CV are needed to maintain a continuous high level of operations. The availability fraction, μ, represents an aggregate measure of this complex aircraft supply process.

The Barrier Logistics Model Completed

The formula for total logistics draw, Y_B, including maintainability and operational factors is given by

$$Y_B = y\left\{(1+\tau)/\mu\left(\{S/(1-2B/C)\} + y_I\right)\right\}$$

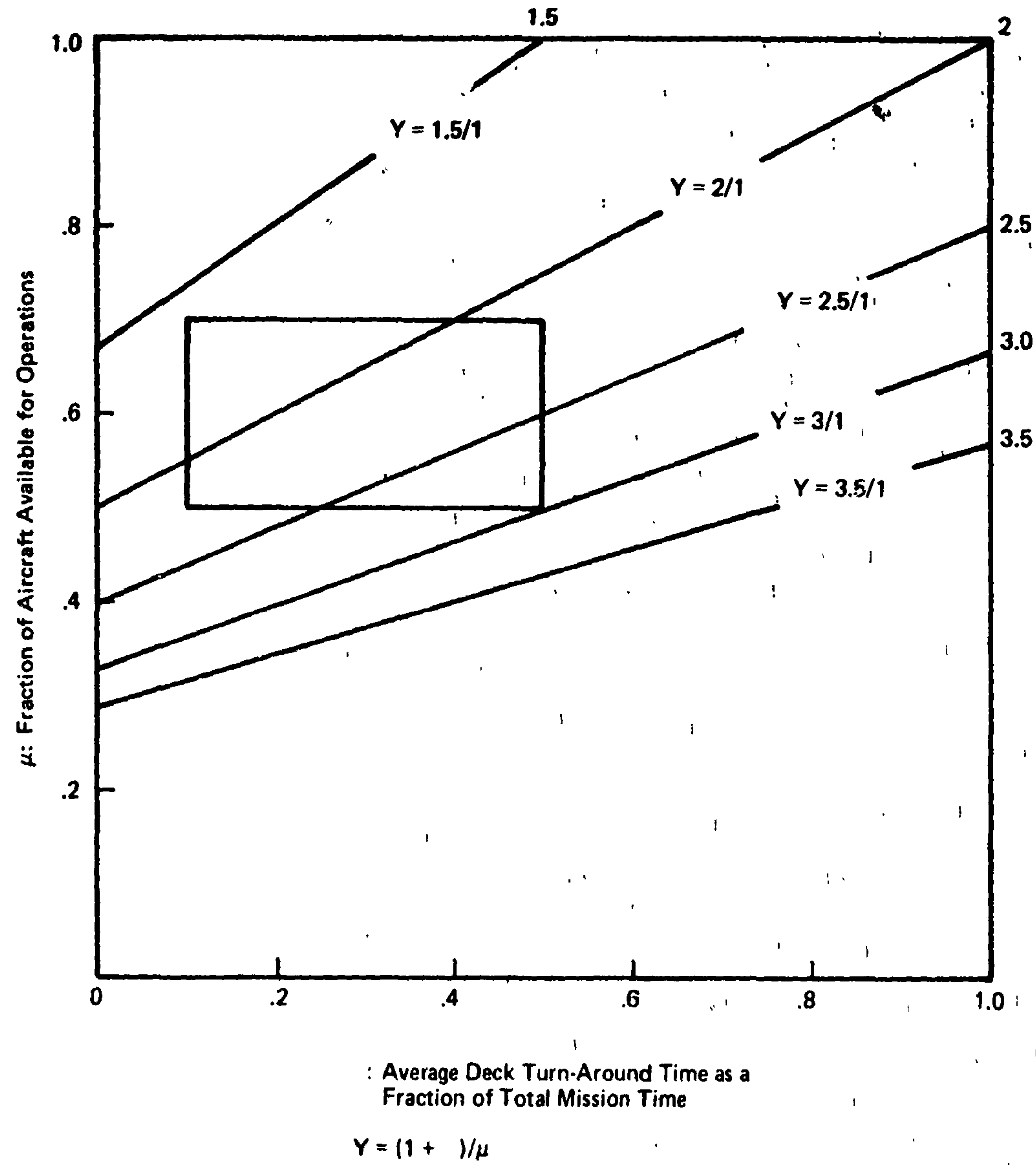


FIGURE 7. LOGISTICS DRAW DUE TO AIRCRAFT MAINTAINABILITY

where {a} denotes the next largest integer to the number a; y denotes the number of aircraft per barrier station; and y_I denotes the increment needed to provide for CV mobility throughout an objective area of given size.

Table 9 displays values of logistics draw, Y_B, for a range of values of station surveillance width, aircraft maintainability, cruise ranges and barrier station range. These data include a suitable allowance, y_I, to permit the CV full mobility up to 100 miles from the center of the barrier in cases when C = 4B and 6B and up to only 50 miles when C = 3B.

Implications of the Air Barrier Logistics Model

These data show that reductions in barrier coverage do not produce reductions in logistics draw comparable to the reductions potentially available from increased surveillance capability, improved maintainability, or range extension. For example, 38 aircraft of 800 mile cruise range, maintenance factor 2.5, surveillance width 200 miles, are needed to tightly cover a barrier at 200 miles ($p \geq .95$) while 30 cover the barrier loosely ($p \geq .75$). An increase in surveillance width from 200 to 300 miles drops the requirement to 25 aircraft for .95 coverage, while a large improvement in the maintainability factor from 2.5 to 1.5 (surveillance width 200 miles) drops the requirement to 23 aircraft.

The data in Table 9 cover a wide range of capabilities and indicate the type improvements necessary to make the establishment of distant ring-type barriers practicable. For example, 25 aircraft of 1200 mile range, maintainability factor 2.5, surveillance width 200 miles provide tight coverage (one aircraft per barrier sector) at 200 mile range, but 50 of these aircraft are needed to extend the barrier to 300 mile range. These 50 can be reduced to 45 by relaxing the barrier coverage to about 75%. An increase in surveillance width from 200 to 300 miles reduces the requirement to 28 aircraft, and a further major increase in maintainability drops the requirement to 17.

If each aircraft on station could destroy three RED recce aircraft in the absence of a fighter escort, RED would need only four aircraft to penetrate one aircraft past the station, five to penetrate two, and six to penetrate three. In addition, BLUE's commitment to the active reconnaissance defense detracts from his capability to meet a following bomber attack. Thus, the methodology permits a comparative evaluation of BLUE's commitment of interceptors and RED's penetration problem. Given an estimate of the quality of RED's targeting intelligence on penetration, and BLUE's reserve interceptor force, the earlier air battle model can be used to measure the severity of the follow-on bomber attack.

TABLE 9

NUMBER OF AIRCRAFT OF GIVEN SURVEILLANCE, MAINTAINABILITY, AND CRUISE RANGE CHARACTERISTICS REQUIRED TO PATROL BARRIERS AT 200, 300, AND 400 MILES

Station Surveillance Width, nmi		200						300						
Maintenance Factor		2.5			1.5			2.5			1.5			
Cruise Range, nmi		3B	4B	6B	3B	4B	6B	3B	4B	6B	3B	4B	6B	
Barrier Range, nmi: Nmi	No. of Stations													No. of Stations
Barrier Coverage $p \geq .95$														
200	6	55	38	25	33	23	15	38	25	18	23	15	11	4
300	9	75	50	38	45	30	23	50	33	25	30	20	15	6
400	12	95	63	48	57	38	29	63	43	30	38	26	18	8
Barrier Coverage $p \geq .75$														
200	5	45	30	23	27	18	14	38	25	18	23	15	11	4
300	8	65	45	33	39	27	20	43	28	23	26	17	14	5
400	10	78	53	38	47	32	23	55	38	28	33	23	17	7

While it is premature to draw conclusions about the efficacy of active defense against air reconnaissance, the large logistics support such a defense requires to achieve only a modest fighting strength at the barrier, suggests that the CTG capabilities for ECM, decoying the RED radar reconnaissance to create false targets, and prompt attack of radiating aircraft that do penetrate, will prove to be more effective in protecting the CTG than attempts to deny, or significantly raise, RED's cost of penetrating to within radar surveillance range of the objective area. However, when RED has few reconnaissance aircraft, active defense against these can be an effective deterrent.